1. $y = 5x^2 + 3x - 1$ at $x = 2$

$y' = 10x + 3$ $\quad 10(2) + 3 = \boxed{23}$

2. $y = 5x^{-3} + 5$ at $x = 3$

$y' = \frac{-15}{x^4}$ $\quad \frac{-15}{3^4} = \boxed{\frac{-5}{27}}$

3. $y = \frac{x+2}{5x-1}$ at $x = 4$

$y' = \frac{(5x-1)(1) - (x+2)(5)}{(5x-1)^2} = \frac{5x - 1 - 5x - 10}{(5x-1)^2}\Big|_{x=4} = \frac{-11}{19^2} = \boxed{\frac{-11}{361}}$

4. $y = x^2 e^x$ at $x = 3$

$y' = x^2 e^x + 2x e^x = xe^x(x+2)\Big|_{x=3} = 3e^3(5) = \boxed{15e^3}$

5. $y = (3x-1)(2^x)$ at $x = 5$

$y' = (3x-1)(2^x)(\ln 2) + 3(2^x) = 2^x\left((3x-1)(\ln 2) + 3\right)\Big|_{x=5}$

$= \boxed{32\left[14\ln 2 + 3\right]}$

6. $y = \frac{e^x}{\ln x}$ at $x = e$

$y' = \frac{(\ln x)(e^x) - \frac{e^x}{x}}{(\ln x)^2}\Bigg|_{x=e} = \frac{(\ln e)(e^e) - \frac{e^e}{e}}{(\ln e)^2} = \boxed{e^e - \frac{e^e}{e}}$

7. $y = (\sin 3x)(\cos 2x)$ at $x = \pi$

$y' = (\sin 3x)(-2\sin 2x) + (3\cos 3x)(\cos 2x) \Big|_{x=\pi}$

$= (0)(0) + (3)(-1)(1) = \boxed{-3}$

8. $y = \ln(x^2)$ at $x = 3$

$y' = \frac{1}{x^2} \cdot 2x = \frac{2}{x} \Big|_{x=3} = \boxed{\frac{2}{3}}$

9. $y = \log_4(x^2 - x - 1)$ at $x = 4$

$y' = \frac{2x-1}{(\ln 4)(x^2-x-1)} \Big|_{x=4} = \frac{7}{(\ln 4)(11)} = \boxed{\frac{7}{11\ln 4}}$

10. $y = \frac{x^2+1}{\cos x}$ at $x = \pi$

$y' = \frac{(\cos x)(2x) - (x^2+1)(-\sin x)}{\cos^2 x} \Big|_{x=\pi} = \frac{(-1)(2\pi) - (\pi^2+1)(0)}{(-1)^2} = \boxed{-2\pi}$

11. $y = \ln(\sin(x^2))$ at $x = \sqrt{\frac{\pi}{6}}$

$y' = \frac{2x\cos(x^2)}{\sin(x^2)} \Big|_{x=\sqrt{\frac{\pi}{6}}} = \frac{2\sqrt{\frac{\pi}{6}}\left(\frac{\sqrt{3}}{2}\right)}{\frac{1}{2}} = 2\sqrt{\frac{\pi}{6}} \cdot \sqrt{3} = \boxed{2\sqrt{\frac{\pi}{2}}}$

12. $y = \tan^{-1}(7x)$ at $x = 2$

$y' = \frac{7}{1+(7x)^2} \Big|_{x=2} = \frac{7}{1+14^2} = \boxed{\frac{7}{197}}$

13. $y = \dfrac{\sin^{-1} 2x}{x^2}$ at $x = \dfrac{1}{4}$

$$y' = \frac{\dfrac{2x^2}{\sqrt{1-4x^2}} - 2x\sin^{-1}(2x)}{x^4}\Bigg|_{x=\frac{1}{4}} = \frac{\dfrac{\frac{1}{8}}{\frac{\sqrt{3}}{2}} - \frac{1}{2}\left(\frac{\pi}{6}\right)}{\frac{1}{256}}$$

$$= \boxed{\left(\frac{1}{4\sqrt{3}} - \frac{\pi}{12}\right)256}$$

14. $y = \sec^{-1}\sqrt{x}$ at $x = 2$

$$y' = \frac{1}{|\sqrt{x}|\sqrt{(\sqrt{x})^2 - 1}} \cdot \frac{1}{2\sqrt{x}} = \frac{1}{2x\sqrt{x-1}}\Big|_{x=2} = \boxed{\frac{1}{4}}$$

15. $y = \ln(e^x + x + 1)$ at $x = 0$

$$y' = \frac{e^x + 1}{e^x + x + 1}\Big|_{x=0} = \frac{2}{2} = \boxed{1}$$

16. $xy + y^3 = x^2 - y^2 + 13$ at $(1, 2)$

$$x\frac{dy}{dx} + y + 3y^2\frac{dy}{dx} = 2x - 2y\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{2x - y}{x + 3y^2 + 2y}\Big|_{(1,2)} = \frac{2(1) - 2}{1 + 12 + 4} = \boxed{0}$$

17. $\sin(xy) + x^2 - y = -\frac{xy}{2} + 4 \quad \text{at } (2, \pi)$

$$\cos(xy)\left[x\frac{dy}{dx} + y\right] + 2x - \frac{dy}{dx} = -\frac{1}{2}\left[x\frac{dy}{dx} + y\right]$$

$$x\cos(xy)\frac{dy}{dx} + y\cos(xy) + 2x - \frac{dy}{dx} = -\frac{1}{2}x\frac{dy}{dx} - \frac{1}{2}y$$

$$\frac{dy}{dx} = \left.\frac{y\cos(xy) + 2x + \frac{1}{2}y}{1 - \frac{1}{2}x - x\cos(xy)}\right|_{(2,\pi)} = \frac{\pi\cos 2\pi + 4 + \frac{\pi}{2}}{1 - 1 - 2\cos 2\pi} = \frac{\frac{3\pi}{2} + 4}{-2}$$

$$= \boxed{-\frac{3\pi}{4} - 2}$$

18. $x^2y + xy^2 = 2xy \quad \text{at } (1, 1)$

$$x^2\frac{dy}{dx} + 2xy + 2xy\frac{dy}{dx} + y^2 = 2x\frac{dy}{dx} + 2y$$

$$\frac{dy}{dx} = \left.\frac{2y - 2xy - y^2}{x^2 + 2xy - 2x}\right|_{(1,1)} = \frac{2 - 2 - 1}{1 + 2 - 2} = \boxed{-1}$$

19. $x\sin y + \tan y = x^2 \quad \text{at } \left(\sqrt{2}, \frac{\pi}{4}\right)$

$$x\cos y\frac{dy}{dx} + \sin y + \sec^2 y\frac{dy}{dx} = 2x$$

$$\frac{dy}{dx} = \left.\frac{2x - \sin y}{x\cos y + \sec^2 y}\right|_{(\sqrt{2}, \frac{\pi}{4})} = \frac{2\sqrt{2} - \frac{\sqrt{2}}{2}}{\sqrt{2}\left(\frac{\sqrt{2}}{2}\right) + 2} = \boxed{\frac{\sqrt{2}}{2}}$$

20. $\frac{x+y}{x-y} = x^2y^2 - 41$ at $(2,3)$

$$\frac{(x-y)\left(1+\frac{dy}{dx}\right) - (x+y)\left(1-\frac{dy}{dx}\right)}{(x-y)^2} = x^2(2y)\frac{dy}{dx} + 2xy^2$$

$$x + x\frac{dy}{dx} - y - y\frac{dy}{dx} - x + x\frac{dy}{dx} - y + y\frac{dy}{dx} = 2x^2y(x-y)^2\frac{dy}{dx} + 2xy^2(x-y)^2$$

$$2x\frac{dy}{dx} - 2y = 2x^2y(x-y)^2\frac{dy}{dx} + 2xy^2(x-y)^2$$

$$\frac{dy}{dx} = \frac{2xy^2(x-y)^2 + 2y}{2x - 2x^2y(x-y)^2}\Bigg|_{(2,3)}$$

$$= \frac{2(2)(9)(1) + 6}{4 - (24)(1)} = \frac{42}{-20} = \boxed{-\frac{21}{10}}$$