1. a) $y = \sin^4(x^3)$ $\quad y = u^4 \quad u = \sin(x^3)$

$u = \sin v \quad v = x^3$

$\frac{dy}{du} = 4u^3 \quad \frac{du}{dv} = \cos v \quad \frac{dv}{dx} = 3x^2$

$y' = \boxed{12x^2 \sin^3(x^3)\cos(x^3)}$

b) $y = x^2 \sin^{-1} x$ $\quad y = uv$ product rule

$$\boxed{y' = \frac{x^2}{\sqrt{1-x^2}} + 2x \sin^{-1} x}$$

c) $y = e^{\tan(x^2-2)}$ $\quad y = e^u \quad u = \tan(x^2-2)$

$u = \tan v \quad v = x^2 - 2$

$\frac{dy}{du} = e^u \quad \frac{du}{dv} = \sec^2 v \quad \frac{dv}{dx} = 2x$

$$\boxed{\frac{dy}{dx} = 2xe^{\tan(x^2-2)}(\sec^2(x^2-2))}$$

d) $y = \ln(7x - e^{2x})$ $\quad y = \ln u \quad u = 7x - e^{2x}$

$\frac{dy}{du} = \frac{1}{u} \quad \frac{du}{dx} = 7 - 2e^{2x}$

$$\boxed{\frac{dy}{dx} = \frac{7-2e^{2x}}{7x - e^{2x}}}$$

e) $y = \sec^{-1}(x^3+1)$ $\quad y = \sec^{-1} u \quad u = x^3 + 1$

$$\frac{dy}{du} = \frac{1}{|x^3+1|\sqrt{(x^3+1)^2 - 1}} \qquad \frac{du}{dx} = 3x^2$$

$$\boxed{\frac{dy}{dx} = \frac{3x^2}{|x^3+1|\sqrt{x^6 + 2x^3}}}$$

f) $f(x) = \sqrt{\cot x} - 2x^{\frac{1}{3}}$

$\sqrt{u} \quad u = \cot x$

$$\boxed{\frac{-\csc^2 x}{2\sqrt{\cot x}} - \frac{2}{3}x^{-\frac{2}{3}}}$$

g) $y = (3x)^{x^2}$ $\quad \ln y = \underbrace{x^2}_{u}\underbrace{\ln(3x)}_{v}$

$$\frac{1}{y}\frac{dy}{dx} = \frac{x^2(3)}{3x} + 2x\ln(3x)$$

$$\boxed{\frac{dy}{dx} = (x + 2x\ln 3x)(3x)^{(x^2)}}$$

h) $y = \dfrac{3x-5}{2x^2+1} = \dfrac{u}{v}$ $\quad \dfrac{(2x^2+1)(3) - (3x-5)(4x)}{(2x^2+1)^2}$

$$= \frac{6x^2+3 - 12x^2 + 20x}{(2x^2+1)^2} = \boxed{\frac{-6x^2+20x+3}{(2x^2+1)^2}}$$

i) $y = \frac{e^x}{\ln x} = \frac{u}{v}$

$$\boxed{\frac{(\ln x)(e^x) - \frac{e^x}{x}}{(\ln x)^2}}$$

j) $y = (e^{\sin x})(x^3 - \cos x)$

$(u)(v)$

$$\boxed{\frac{dy}{dx} = (e^{\sin x})(3x^2 + \sin x) + (x^3 - \cos x)(e^{\sin x})(\cos x)}$$

k) $y = \tan^{-1}(\ln(x^2))$ $\quad y = \tan^{-1} u \quad u = \ln x^2$

$u = \ln v \quad v = x^2$

$$\left(\frac{1}{1+u^2}\right)\left(\frac{1}{v}\right)(2x) = \frac{2x}{(1+(\ln x^2)^2)(x^2)}$$

$$= \boxed{\frac{2}{x(1+(\ln x^2)^2)}}$$

l) $y = 3^{(e^x)}$

$y = 3^u \quad u = e^x$

$\downarrow \frac{dy}{du} \qquad \frac{du}{dx}$

$$3^u(\ln 3)(e^x) = \boxed{3^{e^x}(\ln 3)(e^x)}$$

m) $y = (\sec^{-1} x)^x \qquad \ln y = x \ln \sec^{-1} x$

$$\frac{1}{y}\frac{dy}{dx} = \left(\frac{x}{\sec^{-1} x}\right)\left(\frac{1}{|x|\sqrt{x^2-1}}\right) + \ln \sec^{-1} x$$

$$\boxed{\frac{dy}{dx} = \left[\frac{x}{|x|\sqrt{x^2-1}\ \sec^{-1} x} + \ln \sec^{-1} x\right](\sec^{-1} x)^x}$$

n) $x^2 - y^2 = xy$

$$2x - 2y\frac{dy}{dx} = x\frac{dy}{dx} + y$$

$$2x - y = (x+2y)\frac{dy}{dx}$$

$$\boxed{\frac{dy}{dx} = \frac{2x-y}{x+2y}}$$

o) $2xy + x^2y^2 = 1$

$$2x\frac{dy}{dx} + 2y + x^2(2y)\frac{dy}{dx} + 2xy^2 = 0$$

$$(2x + 2x^2y)\frac{dy}{dx} = -2xy^2 - 2y$$

$$\boxed{\frac{dy}{dx} = \frac{-xy^2 - y}{x + x^2y}}$$

p) $x\cos(y^2) = 4y - 2$

$$x(-\sin(y^2))(2y)\frac{dy}{dx} + \cos(y^2) = 4\frac{dy}{dx}$$

$$\cos y^2 = (4 + 2xy\sin y^2)\frac{dy}{dx}$$

$$\boxed{\frac{\cos y^2}{4 + 2xy\sin y^2} = \frac{dy}{dx}}$$

q) $\tan(xy-1) = 2x + y$

$$\sec^2(xy-1)\left[x\frac{dy}{dx} + y\right] = 2 + \frac{dy}{dx}$$

$$x\sec^2(xy-1)\frac{dy}{dx} + y\sec^2(xy-1) = 2 + \frac{dy}{dx}$$

$$y\sec^2(xy-1) - 2 = \frac{dy}{dx}\left(1 - x\sec^2(xy-1)\right)$$

$$\boxed{\frac{dy}{dx} = \frac{y\sec^2(xy-1) - 2}{1 - x\sec^2(xy-1)}}$$

r) $y = (5-x^2)^\pi$ $\quad y = u^\pi \quad u = 5 - x^2$

$$\boxed{\pi(5-x^2)^{\pi-1}(-2x)}$$

s) $y = (x^2 + \ln x)^4$ $\quad y = u^4 \quad u = x^2 + \ln x$

$$\boxed{4(x^2 + \ln x)^3\left(2x + \frac{1}{x}\right)}$$

t) $y = \ln(\ln x^2)$ $\quad y = \ln u \quad u = \ln x^2 = 2\ln x$

$$\left(\frac{1}{\ln x^2}\right)\left(\frac{2}{x}\right) = \frac{1\,(2)}{(2\ln x)x} = \boxed{\frac{1}{x\ln x}}$$

u) $y = \sec(x\sin x)$ $\quad y = \sec u \quad u = x\sin x$

$$\boxed{\sec(x\sin x)\tan(x\sin x)\left[x\cos x + \sin x\right]}$$

v) $y = \sqrt{e^x - \cos(x^2)}$ $\quad y = \sqrt{u} \quad u = e^x - \cos(x^2)$

$\quad\quad -\cos u \quad u = x^2$

$$\frac{1}{2\sqrt{e^x - \cos(x^2)}}\left(e^x + 2x\sin(x^2)\right)$$

$$= \boxed{\frac{e^x + 2x\sin x^2}{2\sqrt{e^x - \cos x^2}}}$$

w) $y = x^{\ln x}$ $\quad \ln y = (\ln x)(\ln x)$

$$\frac{1}{y}\frac{dy}{dx} = \frac{\ln x}{x} + \frac{\ln x}{x}$$

$$\boxed{\frac{dy}{dx} = \frac{2\ln x}{x}\left(x^{\ln x}\right)}$$

x) $y = (\sin 4x)(\tan(x^3 - 1))$ $\quad \boxed{(\sin 4x)\sec^2(x^3-1)[3x^2] + 4\cos 4x\tan(x^3-1)}$

$\quad\quad u \quad\quad v$

y) $y = x^2(e^{e^x})$ $\quad \boxed{x^2(e^{e^x})(e^x) + 2xe^{e^x}}$

$\quad\quad u \quad v$

z) $y = \sqrt{\ln\sqrt{x}}$ $\quad y = \sqrt{u} \quad u = \ln\sqrt{x} = \ln x^{\frac{1}{2}} = \frac{1}{2}\ln x$

$$\frac{1}{2\sqrt{\ln\sqrt{x}}} \cdot \frac{1}{2x} = \boxed{\frac{1}{4x\sqrt{\ln\sqrt{x}}}}$$

2. a) $y = x^3 - 5x^2 - x - 1$ at $x = -1$

$$y' = 3x^2 - 10x - 1 \Big|_{x=-1} = 3(-1)^2 - 10(-1) - 1$$

$$= 3 + 10 - 1 = \boxed{12}$$

b) $y = x^3 \ln x$ at $x = 1$

$$y' = \frac{x^3}{x} + 3x^2 \ln x = x^2 + 3x^2 \ln x \Big|_{x=1} = 1^2 + 3(1^2)\ln 1$$

$$= 1^2 + 0 = \boxed{1}$$

c) $y = \sin x \cos 2x$ at $x = \frac{\pi}{6}$

$$y' = \Big[\sin x(-2\sin 2x) + \cos x \cos 2x\Big]_{x=\frac{\pi}{6}}$$

$$= \left(\sin\frac{\pi}{6}\right)\left(-2\sin\frac{\pi}{3}\right) + \cos\frac{\pi}{6}\cos\frac{\pi}{3}$$

$$= \left(\frac{1}{2}\right)\left(-\frac{2\sqrt{3}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) = \frac{-2\sqrt{3}}{4} + \frac{\sqrt{3}}{4} = \boxed{\frac{-\sqrt{3}}{4}}$$

d) $y = \frac{\tan x}{x}$ at $x = \frac{\pi}{4}$

$$y' = \frac{x\sec^2 x - \tan x}{x^2}\Bigg|_{x=\frac{\pi}{4}} = \frac{\frac{\pi}{4}\sec^2\left(\frac{\pi}{4}\right) - \tan\frac{\pi}{4}}{\left(\frac{\pi}{4}\right)^2}$$

$$= \frac{\frac{\pi}{4}(2) - 1}{\frac{\pi^2}{16}} = \boxed{\frac{\frac{\pi}{2} - 1}{\frac{\pi^2}{16}}}$$

e) $y = \tan^{-1}(x^2)$ $\quad y = \tan^{-1} u \quad u = x^2$

at $x = \frac{1}{3}$

$$\left.\frac{2x}{1+x^4}\right|_{x=} = \frac{2\left(\frac{1}{3}\right)}{1+\left(\frac{1}{3}\right)^4} = \frac{\frac{2}{3}}{1+\frac{1}{81}}$$

$$= \frac{\frac{2}{3}}{\frac{82}{81}} = \frac{2(81)}{3(82)} = \boxed{\frac{27}{41}}$$

f) $y = \sqrt{x^2-3}$ at $x = 2$ $\quad y = \sqrt{u} \quad u = x^2 - 3$

$$\left.\frac{1(2x)}{2\sqrt{x^2-3}}\right|_{x=2} = \frac{2(2)}{2\left(\sqrt{2^2-3}\right)} = \frac{2}{\sqrt{1}} = \boxed{2}$$

g) $x^2y^3 - x - y = 1$ at $(2,1)$

$$x^2(3y^2)\frac{dy}{dx} + 2xy^3 - 1 - \frac{dy}{dx} = 0$$

$$\frac{dy}{dx}\left(3x^2y^2 - 1\right) = 1 - 2xy^3$$

$$\frac{dy}{dx} = \left.\frac{1-2xy^3}{3x^2y^2-1}\right|_{(2,1)} = \frac{1-2(2)(1^3)}{3(2^2)(1^2)-1}$$

$$= \frac{1-4}{12-1} = \boxed{\frac{-3}{11}}$$

h) $x^3 + y^2 = xy + 1$ at $(-1, 1)$

$$3x^2 + 2y\frac{dy}{dx} = x\frac{dy}{dx} + y$$

$$3x^2 - y = \frac{dy}{dx}(x - 2y)$$

$$\frac{dy}{dx} = \left.\frac{3x^2 - y}{x - 2y}\right|_{(-1,1)} = \frac{3(-1)^2 - 1}{(-1) - 2(1)}$$

$$= \frac{3 - 1}{-1 - 2} = \frac{2}{-3} = \boxed{-\frac{2}{3}}$$

i) $x\cos y + y\cos x = y^2 + x$

$$x(-\sin y)\frac{dy}{dx} + \cos y + y(-\sin x) + (\cos x)\frac{dy}{dx} = 2y\frac{dy}{dx} + 1$$

$$(\cos x - x\sin y - 2y)\frac{dy}{dx} = 1 + y\sin x - \cos y$$

$$\frac{dy}{dx} = \left.\frac{1 + y\sin x - \cos y}{\cos x - x\sin y - 2y}\right|_{(\pi, 0)}$$

$$= \frac{1 + 0(\sin\pi) - \cos 0}{\cos\pi - \pi\sin 0 - 2(0)} = \frac{1 + 0 - 1}{-1 - 0 - 0} = \frac{0}{1} = \boxed{0}$$