

Chain Rule

1. $y = \sin 5x$ $y = \sin u$ $u = 5x$

$$\frac{dy}{du} = \cos u \quad \frac{du}{dx} = 5$$

$$\frac{dy}{dx} = \boxed{5 \cos 5x}$$

2. $y = \cos 4x$ $y = \cos u$ $u = 4x$

$$\frac{dy}{du} = -\sin u \quad \frac{du}{dx} = 4$$

$$\frac{dy}{dx} = \boxed{-4 \sin 4x}$$

3. $y = \tan\left(\frac{1}{7}x\right)$ $y = \tan u$ $u = \frac{1}{7}x$

$$\frac{dy}{du} = \sec^2 u \quad \frac{du}{dx} = \frac{1}{7}$$

$$\frac{dy}{dx} = \boxed{\frac{1}{7} \sec^2\left(\frac{1}{7}x\right)}$$

4. $y = \sec(x^2)$ $y = \sec u$ $u = x^2$

$$\frac{dy}{du} = \sec u \tan u \quad \frac{du}{dx} = 2x$$

$$\frac{dy}{dx} = 2x \sec(x^2) \tan(x^2)$$

5. $y = \cot(5x^3)$ $y = \cot u$ $u = 5x^3$

$$\frac{dy}{du} = -\csc^2 u \quad \frac{du}{dx} = 15x^2$$

$$\boxed{\frac{dy}{dx} = -15x^2 \csc^2(5x^3)}$$

$$6. y = \csc 10x$$

$$y = \csc u \quad u = 10x$$

$$\frac{dy}{du} = -\csc u \cot u \quad \frac{du}{dx} = 10$$

$$\frac{dy}{dx} = -10 \csc 10x \cot 10x$$

$$7. y = \cos(x^3 + x^2)$$

$$y = \cos u \quad u = x^3 + x^2$$

$$\frac{dy}{du} = -\sin u \quad \frac{du}{dx} = 3x^2 + 2x$$

$$\frac{dy}{dx} = -(3x^2 + 2x) \sin(x^3 + x^2)$$

$$8. y = \sin(3x^5 - 7x + 1)$$

$$y = \sin u \quad u = 3x^5 - 7x + 1$$

$$\frac{dy}{du} = \cos u \quad \frac{du}{dx} = 15x^4 - 7$$

$$\frac{dy}{dx} = (15x^4 - 7) \cos(3x^5 - 7x + 1)$$

$$9. y = e^{5x}$$

$$y = e^u \quad u = 5x$$

$$\frac{dy}{du} = e^u \quad \frac{du}{dx} = 5$$

$$\frac{dy}{dx} = 5e^{5x}$$

$$10. y = e^{-\frac{1}{2}x}$$

$$y = e^u \quad u = -\frac{1}{2}x$$

$$\frac{dy}{du} = e^u \quad \frac{du}{dx} = -\frac{1}{2}$$

$$\frac{dy}{dx} = -\frac{1}{2}e^{-\frac{1}{2}x}$$

$$11. y = e^{5x^3}$$

$$y = e^u \quad u = 5x^3$$

$$\frac{dy}{du} = e^u \quad \frac{du}{dx} = 15x^2$$

$$\frac{dy}{dx} = 15x^2 e^{5x^3}$$

$$12. y = 5^{8x}$$

$$y = 5^u \quad u = 8x$$

$$\frac{dy}{du} = 5^u \ln 5 \quad \frac{du}{dx} = 8$$

$$\frac{dy}{dx} = 8 \ln(5) 5^{8x}$$

$$13. y = \pi^{(x^2)}$$

$$y = \pi^u \quad u = x^2$$

$$\frac{dy}{du} = \pi^u \ln \pi \quad \frac{du}{dx} = 2x$$

$$\frac{dy}{dx} = 2x \ln(\pi) \pi^{x^2}$$

$$14. y = 10^{x^2-3}$$

$$y = 10^u \quad u = x^2 - 3$$

$$\frac{dy}{du} = 10^u \ln 10 \quad \frac{du}{dx} = 2x$$

$$\frac{dy}{dx} = 2x \ln(10) 10^{x^2-3}$$

$$15. y = \ln(x^3)$$

$$y = \ln u \quad u = x^3$$

$$\frac{dy}{du} = \frac{1}{u} \quad \frac{du}{dx} = 3x^2$$

$$\frac{dy}{dx} = \frac{3x^2}{x^3} = \boxed{\frac{3}{x}}$$

$$16. y = \ln(\sin x)$$

$$y = \ln u \quad u = \sin x$$

$$\frac{dy}{du} = \frac{1}{u} \quad \frac{du}{dx} = \cos x$$

$$\frac{dy}{dx} = \frac{\cos x}{\sin x} = \boxed{\cot x}$$

$$17. y = \ln(x - \tan x)$$

$$y = \ln u \quad u = x - \tan x$$

$$\frac{dy}{du} = \frac{1}{u} \quad \frac{du}{dx} = 1 - \sec^2 x$$

$$\frac{dy}{dx} = \frac{1 - \sec^2 x}{x - \tan x} = -\frac{\tan^2 x}{x - \tan x}$$

$$18. y = \log_7(x^3 - 1)$$

$$y = \log_7 u \quad u = x^3 - 1$$

$$\frac{dy}{du} = \frac{1}{u \ln 7} \quad \frac{du}{dx} = 3x^2$$

$$\frac{dy}{dx} = \frac{3x^2}{(\ln 7)(x^3 - 1)}$$

$$19. y = \log(x^2 + x + 1)$$

$$y = \log u \quad u = x^2 + x + 1$$

$$\frac{dy}{du} = \frac{1}{u \ln 10} \quad \frac{du}{dx} = 2x + 1$$

$$\frac{dy}{dx} = \frac{2x + 1}{(x^2 + x + 1) \ln 10}$$

$$20. y = \log_3(3x^3 - x)$$

$$y = \log_3 u \quad u = 3x^3 - x$$

$$\frac{dy}{du} = \frac{1}{u \ln 3} \quad \frac{du}{dx} = 9x^2 - 1$$

$$\frac{dy}{dx} = \frac{9x^2 - 1}{(3x^3 - x) \ln 3}$$