

1. $y = x^2 + \cos 5x$ $\rightarrow$ $\cos u$ $u = 5x$

$$\frac{dy}{dx} = 2x - 5 \sin 5x$$

2. $y = \sin(x^3 + e^x)$ $\sin u$ $u = x^3 + e^x$

$$\frac{dy}{dx} = \cos(x^3 + e^x) [3x^2 + e^x]$$

3. $y = \sin^3 x$ u^3 $u = \sin x$

$$\frac{dy}{dx} = 3 \sin^2 x \cos x$$

4. $y = 7x - 3\cos(x^3)$ $3\cos u$ $u = x^3$

$$\frac{dy}{dx} = 7 + 3\sin(x^3) [3x^2] = 7 + 9x^2 \sin(x^3)$$

5. $y = \frac{x^2+3}{x^2-3}$

$$\frac{dy}{dx} = \frac{(x^2-3)(2x) - (x^2+3)(2x)}{(x^2-3)^2} = \frac{2x^3 - 6x - 2x^3 - 6x}{(x^2-3)^2}$$

$$= \frac{-12x}{(x^2-3)^2}$$

6. $y = x^5 e^x$

$$\frac{dy}{dx} = x^5 e^x + 5x^4 e^x = x^4 e^x (x+5)$$

7. $y = \sqrt{x^3 - 1}$ $\sqrt{u}$ $u = x^3 - 1$

$$\frac{dy}{dx} = \frac{3x^2}{2\sqrt{x^3 - 1}}$$

8. $y = e^{\sqrt{x}}$ e^u $u = \sqrt{x}$

$$\frac{dy}{dx} = \frac{e^{\sqrt{x}}}{2\sqrt{x}}$$

9. $y = x^2 \tan(x^2)$ $\tan u$ $u = x^2$

$$\frac{dy}{dx} = x^2 [\sec^2(x^2) [2x]] + 2x \tan(x^2)$$

$$= 2x [x^2 \sec^2(x^2) + \tan(x^2)]$$

10. $y = \sqrt{x^3 + x - 11}$ $\sqrt{u}$ $u = x^3 + x - 11$

$$\frac{dy}{dx} = \frac{3x^2 + 1}{2\sqrt{x^3 + x - 11}}$$

11. $y = \frac{x^2 + 1}{x^3 - 1}$ $\frac{dy}{dx} = \frac{(x^3 - 1)(2x) - (x^2 + 1)(3x^2)}{(x^3 - 1)^2}$

$$= \frac{2x^4 - 2x - 3x^4 - 3x^2}{(x^3 - 1)^2} = \frac{-x^4 - 3x^2 - 2x}{(x^3 - 1)^2}$$

$$\begin{aligned}
 12. \quad y &= \frac{3 + \sec x}{\tan x} & \frac{dy}{dx} &= \frac{\tan x (\sec x \tan x) - (3 + \sec x)(\sec^2 x)}{\tan^2 x} \\
 &= \frac{\sec x \tan^2 x - 3 \sec^2 x - \sec^3 x}{\tan^2 x} & &= \frac{\sec x (\tan^2 x - 3 \sec x - \sec^2 x)}{\tan^2 x} \\
 &= \frac{\sec x (-1 - 3 \sec x)}{\tan^2 x} & &= \boxed{\frac{-\sec x (1 + 3 \sec x)}{\tan^2 x}}
 \end{aligned}$$

$$\begin{aligned}
 13. \quad y &= \sqrt{x^2 - e^x} (\sin x) \\
 \frac{dy}{dx} &= \sqrt{x^2 - e^x} (\cos x) + \frac{(\sin x)(2x - e^x)}{2\sqrt{x^2 - e^x}}
 \end{aligned}$$

$$= \boxed{\frac{2(x^2 - e^x) \cos x + \sin x (2x - e^x)}{2\sqrt{x^2 - e^x}}}$$

$$14. \quad y = \cot^3 x \quad u^3 \quad u = \cot x$$

$$\frac{dy}{dx} = 3 \cot^2 x (-\csc^2 x) = \boxed{-3 \cot^2 x \csc^2 x}$$

$$15. \quad y = 3^x \underbrace{\csc(x^3)}_{\csc u} \quad u = x^3$$

$$\frac{dy}{dx} = 3^x (-\csc x^3 \cot x^3)(3x^2) + 3^x (\ln 3) \csc x^3$$

$$= \boxed{3^x \csc x^3 [\ln 3 - 3x^2 \cot x^3]}$$

16. $y = \sqrt[3]{x - \sin^2 x}$ $\sqrt[3]{u}$ $u = x - \sin^2 x$

$$\frac{dy}{dx} = \frac{1 - 2\sin x \cos x}{3(x - \sin^2 x)^{\frac{2}{3}}}$$

17. $y = e^{\sin^5 x}$ e^u $u = \sin^5 x$
 $\hookrightarrow u = v^5$ $v = \sin x$

$$\frac{dy}{dx} = (e^{\sin^5 x})(5\sin^4 x)(\cos x)$$

18. $y = \cos(e^{4x})$ $\cos u$ $u = e^{4x}$
 $\hookrightarrow u = e^v$ $v = 4x$

$$\frac{dy}{dx} = -\sin(e^{4x})[e^{4x}](4) = \boxed{-4e^{4x} \sin(e^{4x})}$$

19. $y = \frac{e^x}{2^x}$ $\frac{dy}{dx} = \frac{2^x e^x - e^x (2^x) \ln 2}{(2^x)^2}$

$$= \boxed{\frac{e^x (1 - \ln 2)}{2^x}}$$

20. $y = (\sin x)(\cos x)e^x = \left(\frac{1}{2} \sin 2x\right)e^x$

$$\frac{dy}{dx} = \left(\frac{1}{2} \sin 2x\right)e^x + e^x \cos 2x$$

$$= \boxed{e^x \left[\frac{1}{2} \sin 2x + \cos 2x \right]}$$

21. $y = \sin(\cos(\tan x))$ $\sin u$ $u = \cos(\tan x)$
 $\hookrightarrow u = \cos v$ $v = \tan x$

$$\frac{dy}{dx} = \cos(\cos(\tan x)) [-\sin(\tan x)] (\sec^2 x)$$

22. $y = e^{(e^x)}$ e^u $u = e^x$

$$\frac{dy}{dx} = e^{(e^x)} e^x = e^{e^x + x}$$

23. $x^2 y^3 + \sin x = y \cos y$

$$x^2 (3y^2 \frac{dy}{dx}) + 2xy^3 + \cos x = y(-\sin y) \frac{dy}{dx} + \cos y \frac{dy}{dx}$$

$$2xy^3 + \cos x = \frac{dy}{dx} (\cos y - y \sin y - 3x^2 y^2)$$

$$\frac{dy}{dx} = \frac{2xy^3 + \cos x}{\cos y - y \sin y - 3x^2 y^2}$$

24. $(\sin x)(\cos y) = x + y - 7$

$$(\sin x)(-\sin y) \frac{dy}{dx} + (\cos x)(\cos y) = 1 + \frac{dy}{dx}$$

$$(\cos x)(\cos y) - 1 = \frac{dy}{dx} (1 + (\sin x)(\sin y))$$

$$\frac{dy}{dx} = \frac{(\cos x)(\cos y) - 1}{1 + (\sin x)(\sin y)}$$

$$25. \sqrt{x+y} = x^2 y^2$$

$$\frac{1}{2\sqrt{x+y}} \left[1 + \frac{dy}{dx} \right] = x^2 (2y) \frac{dy}{dx} + 2xy^2$$

$$1 + \frac{dy}{dx} = 4x^2 y \sqrt{x+y} \frac{dy}{dx} + 4xy^2 \sqrt{x+y}$$

$$1 - 4xy^2 \sqrt{x+y} = \frac{dy}{dx} (4x^2 y \sqrt{x+y} - 1)$$

$$\boxed{\frac{dy}{dx} = \frac{1 - 4xy^2 \sqrt{x+y}}{4x^2 y \sqrt{x+y} - 1}}$$

$$26. e^{xy} - x = y^3$$

$$e^{xy} \left[x \frac{dy}{dx} + y \right] - 1 = 3y^2 \frac{dy}{dx}$$

$$xe^{xy} \frac{dy}{dx} + ye^{xy} - 1 = 3y^2 \frac{dy}{dx}$$

$$ye^{xy} - 1 = \frac{dy}{dx} (3y^2 - xe^{xy})$$

$$\boxed{\frac{dy}{dx} = \frac{ye^{xy} - 1}{3y^2 - xe^{xy}}}$$

$$27. x - xy = y^2 + y^3$$

$$1 - x \frac{dy}{dx} - y = 2y \frac{dy}{dx} + 3y^2 \frac{dy}{dx}$$

$$1 - y = \frac{dy}{dx} (x + 2y + 3y^2)$$

$$\boxed{\frac{dy}{dx} = \frac{1 - y}{x + 2y + 3y^2}}$$

$$28. x \sin y - y \cos x = 10$$

$$x \cos y \frac{dy}{dx} + \sin y + y \sin x - \cos x \frac{dy}{dx} = 0$$

$$\sin y + y \sin x = \frac{dy}{dx} (\cos x - x \cos y)$$

$$\boxed{\frac{dy}{dx} = \frac{\sin y + y \sin x}{\cos x - x \cos y}}$$

$$29. e^x e^y - xy = 2 - \cos x$$

$$e^x e^y \frac{dy}{dx} + e^x e^y - x \frac{dy}{dx} - y = \sin x$$

$$\frac{dy}{dx} (e^x e^y - x) = y + \sin x - e^x e^y$$

$$\boxed{\frac{dy}{dx} = \frac{y + \sin x - e^x e^y}{e^x e^y - x}}$$

$$30. xy^2 - x^3 y^4 + x^5 y^6 = 7$$

$$2xy \frac{dy}{dx} + y^2 - 4x^3 y^3 \frac{dy}{dx} - 3x^2 y^4 + 6x^5 y^5 \frac{dy}{dx} + 5x^4 y^6 = 0$$

$$\frac{dy}{dx} (2xy - 4x^3 y^3 + 6x^5 y^5) = 3x^2 y^4 - y^2 - 5x^4 y^6$$

$$\frac{dy}{dx} = \frac{3x^2 y^4 - y^2 - 5x^4 y^6}{2xy - 4x^3 y^3 + 6x^5 y^5} = \boxed{\frac{3x^2 y^3 - y - 5x^4 y^5}{2x - 4x^3 y^2 + 6x^5 y^4}}$$