

$$1. \quad x^2 + y^2 = x - 2$$

$$2x + 2y \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1 - 2x}{2y}$$

$$\frac{d^2y}{dx^2} = \frac{2y(-2) - (1-2x)2 \frac{dy}{dx}}{(2y)^2}$$

$$= \frac{-4y - (1-2x)(2) \left[\frac{1-2x}{2y} \right]}{4y^2}$$

$$2. \quad xy + x^3 - y^2 = 1$$

$$x \frac{dy}{dx} + y + 3x^2 - 2y \frac{dy}{dx} = 0$$

$$3x^2 + y = (2y - x) \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{3x^2 + y}{2y - x}$$

$$\frac{d^2y}{dx^2} = \frac{(2y - x)(6x + \frac{dy}{dx}) - (3x^2 + y)(2 \frac{dy}{dx} - 1)}{(2y - x)^2}$$

$$= \frac{(2y - x)(6x + \frac{3x^2 + y}{2y - x}) - (3x^2 + y)(2(\frac{3x^2 + y}{2y - x}) - 1)}{(2y - x)^2}$$

$$3. \quad x^2 y^2 + xy + 1 = x + y$$

$$x^2 \left(2y \frac{dy}{dx} \right) + 2xy^2 + x \frac{dy}{dx} + y = 1 + \frac{dy}{dx}$$

$$2xy^2 + y - 1 = \frac{dy}{dx} (1 - x - 2x^2 y)$$

$$\frac{dy}{dx} = \frac{2xy^2 + y - 1}{1 - x - 2x^2 y}$$

$$\frac{d^2 y}{dx^2} = \frac{(1 - x - 2x^2 y) \left(2x \left(2y \frac{dy}{dx} \right) + 2y^2 + \frac{dy}{dx} \right) - (2xy^2 + y - 1) \left(-1 - 2x^2 \frac{dy}{dx} - 4xy \right)}{(1 - x - 2x^2 y)^2}$$

$$= \frac{(1 - x - 2x^2 y) \left(4xy \left(\frac{2xy^2 + y - 1}{1 - x - 2x^2 y} \right) + 2y^2 + \frac{2xy^2 + y - 1}{1 - x - 2x^2 y} \right) - (2xy^2 + y - 1) \left(-1 - 2x^2 \left(\frac{2xy^2 + y - 1}{1 - x - 2x^2 y} \right) - 4xy \right)}{(1 - x - 2x^2 y)^2}$$