Use implicit differentiation to find $\frac{dy}{dx}$.

a. $4x^3 - 2y^3 = x$

$$12x^2 - 6y^2 \frac{dy}{dx} = 1$$

$$\boxed{\frac{dy}{dx} = \frac{1 - 12x^2}{-6y^2}}$$

b. $x^2 + \sqrt{xy} = 7$

$$2x + \frac{1}{2\sqrt{xy}}\left(x\frac{dy}{dx} + y\right) = 0$$

$y = \sqrt{u}$

$u = xy$

$$\frac{1}{2\sqrt{xy}} \cdot D_x[xy]$$

$$\frac{x\frac{dy}{dx} + y}{2\sqrt{xy}} = -2x$$

$$\rightarrow \boxed{\frac{dy}{dx} = \frac{-4x\sqrt{xy} - y}{x}}$$

c. $\sin^2(3y) = x + y - 1$

$y = u^2$

$u = \sin(3y)$

$$2\sin(3y)(3\cos(3y))\frac{dy}{dx} = 1 + \frac{dy}{dx} \rightarrow \frac{dy}{dx}\left[6\sin(3y)\cos(3y) - 1\right] = 1$$

$$\boxed{\frac{dy}{dx} = \frac{1}{6\sin(3y)\cos(3y) - 1}}$$

d. $y^2 = x\cos(y)$

$$2y\frac{dy}{dx} = -x\sin(y)\frac{dy}{dx} + \cos(y)$$

$$\frac{dy}{dx}\left[2y + x\sin(y)\right] = \cos(y)$$

$$\boxed{\frac{dy}{dx} = \frac{\cos(y)}{2y + x\sin(y)}}$$

e. $5x^2 + 2x^2y + y^2 = 3$

$$10x + 2x^2\frac{dy}{dx} + 4xy + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}\left[2x^2 + 2y\right] = -10x - 4xy$$

$$\frac{dy}{dx} = \frac{-10x - 4xy}{2x^2 + 2y}$$

$$= \boxed{\frac{-5x - 2xy}{x^2 + y}}$$

A *Folium of Descartes* has the equation $x^3 + y^3 - 9xy = 0$. Find the slope of the tangent line to the curve at the point $\left(\frac{9}{2}, \frac{9}{2}\right)$.

$$3x^2 + 3y^2 \frac{dy}{dx} - \left(9x \frac{dy}{dx} + 9y\right) = 0$$

$$3x^2 - 9y = 9x \frac{dy}{dx} - 3y^2 \frac{dx}{dx}$$

$$\frac{dy}{dx} = \frac{3x^2 - 9y}{9x - 3y^2} = \frac{x^2 - 3y}{3x - y^2} \Bigg|_{\left(\frac{9}{2}, \frac{9}{2}\right)}$$

$$\frac{\frac{81}{4} - \frac{27}{2}}{\frac{27}{2} - \frac{81}{4}} = \boxed{-1}$$

An *Oval of Cassini* has the equation $(x^2 + y^2 + 4)^2 - 16x^2 = 36$. Find the slope of the tangent line to the curve at the point $(2, \sqrt{2})$.

$$= x^4 + 2x^2y^2 + 8x^2 + 8y^2 + y^4 + 16 - 16x^2 = 36$$

$$= x^4 + 2x^2y^2 - 8x^2 + 8y^2 + y^4 = 20$$

so $$4x^3 + 2x^2(2y)\frac{dy}{dx} + 4x(y^2) - 16x + 16y\frac{dy}{dx} + 4y^3\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}\left[4x^2y + 16y + 4y^3\right] = 16x - 4x^3 - 4xy^2$$

$$\frac{dy}{dx} = \frac{4x - x^3 - xy^2}{x^2y + 4y + y^3}\Bigg|_{(2,\sqrt{2})} = \frac{8 - 8 - 4}{4\sqrt{2} + 4\sqrt{2} + 2\sqrt{2}}$$

$$= \frac{-4}{10\sqrt{2}} = \boxed{\frac{-2}{5\sqrt{2}} \text{ OR } \frac{-\sqrt{2}}{5}}$$

Find the slope of the tangent line to the given curve at the given point:

a. $xy + 16 = 0$; $(2, -8)$

$$x \frac{dy}{dx} + y = 0$$

$$\frac{dy}{dx} = \frac{-y}{x} \Big|_{(2,-8)} = \frac{8}{2} = \boxed{4}$$

b. $y^2 - 4x^2 = 5$; $(-1,3)$

$$2y \frac{dx}{dx} - 8x = 0$$

$$\frac{dy}{dx} = \frac{8x}{2y} = \frac{4x}{y} \Big|_{(-1,3)} = \frac{-4}{3} = -\frac{4}{3}$$

$x^2 y + \sin(y) = 2\pi; (1,2\pi)$

$$x^2 \frac{dy}{dx} + 2xy + \cos(y)\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}\left[x^2 + \cos(y)\right] = -2xy$$

$$\frac{dy}{dx} = \frac{-2xy}{x^2 + \cos(y)}\Bigg|_{(1,2\pi)}$$

$$= \frac{-4\pi}{1+1} = \boxed{-2\pi}$$