

$$1. \quad xy^2 + \cos y = x^3 + 2$$

$$x(2y) \frac{dy}{dx} + y^2 - \sin y \frac{dy}{dx} = 3x^2$$

$$\frac{dy}{dx} (2xy - \sin y) = 3x^2 - y^2$$

$$\boxed{\frac{dy}{dx} = \frac{3x^2 - y^2}{2xy - \sin y}}$$

$$2. \quad xy^2 - \sin y = 3x + x^4$$

$$x(2y) \frac{dy}{dx} + y^2 - \cos y \frac{dy}{dx} = 3 + 4x^3$$

$$\frac{dy}{dx} (2xy - \cos y) = 3 + 4x^3 - y^2$$

$$\boxed{\frac{dy}{dx} = \frac{3 + 4x^3 - y^2}{2xy - \cos y}}$$

$$3. \quad x \cos y + y^2 = xy$$

$$x(-\sin y) \frac{dy}{dx} + \cos y + 2y \frac{dy}{dx} = x \frac{dy}{dx} + y$$

$$-x \sin y \frac{dy}{dx} + 2y \frac{dy}{dx} - x \frac{dy}{dx} = y - \cos y$$

$$\frac{dy}{dx} (2y - x - x \sin y) = y - \cos y$$

$$\boxed{\frac{dy}{dx} = \frac{y - \cos y}{2y - x - x \sin y}}$$

$$4. \quad x^2 y - x = \tan y$$

$$x^2 \frac{dy}{dx} + 2xy - 1 = \sec^2 y \frac{dy}{dx}$$

$$2xy - 1 = \sec^2 y \frac{dy}{dx} - x^2 \frac{dy}{dx}$$

$$2xy - 1 = \frac{dy}{dx} (\sec^2 y - x^2)$$

$$\boxed{\frac{dy}{dx} = \frac{2xy - 1}{\sec^2 y - x^2}}$$