

Limits of the Difference Quotient

$$1) \begin{aligned} & f(x) = 3x - 5 \\ & \lim_{h \rightarrow 0} \frac{3(x+h) - 5 - (3x - 5)}{h} = \lim_{h \rightarrow 0} \frac{3x + 3h - 5 - 3x + 5}{h} = \lim_{h \rightarrow 0} \frac{3h}{h} \\ & = \lim_{h \rightarrow 0} 3 = \boxed{3} \end{aligned}$$

$$2) \begin{aligned} & f(x) = x^2 - 2 \\ & \lim_{h \rightarrow 0} \frac{(x+h)^2 - 2 - (x^2 - 2)}{h} = \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - 2 - x^2 + 2}{h} \\ & = \lim_{h \rightarrow 0} \frac{2xh + h^2}{h} = \lim_{h \rightarrow 0} 2x + h = \boxed{2x} \end{aligned}$$

$$3) \begin{aligned} & f(x) = 2x^2 - x + 2 \\ & \lim_{h \rightarrow 0} \frac{2(x+h)^2 - (x+h) + 2 - (2x^2 - x + 2)}{h} \\ & = \lim_{h \rightarrow 0} \frac{2(x^2 + 2xh + h^2) - x - h + 2 - 2x^2 + x - 2}{h} \\ & = \lim_{h \rightarrow 0} \frac{2x^2 + 4xh + 2h^2 - x - h + 2 - 2x^2 + x - 2}{h} \\ & = \lim_{h \rightarrow 0} \frac{4xh + 2h^2 - h}{h} = \lim_{h \rightarrow 0} 4x + 2h - 1 = \boxed{4x - 1} \end{aligned}$$

$$4) \begin{aligned} & f(x) = \frac{1}{2x+1} \\ & \lim_{h \rightarrow 0} \frac{\frac{1}{2(x+h)+1} - \frac{1}{2x+1}}{h} = \lim_{h \rightarrow 0} \frac{\frac{2x+1}{[2(x+h)+1](2x+1)} - \frac{2(x+h)+1}{[2(x+h)+1](2x+1)}}{h} \\ & = \lim_{h \rightarrow 0} \frac{2x+1 - 2x-2h-1}{h[2(x+h)+1](2x+1)} = \lim_{h \rightarrow 0} \frac{-2h}{h(2x+1)[2(x+h)+1]} \\ & = \lim_{h \rightarrow 0} \frac{-2}{(2x+1)[2(x+h)+1]} = \boxed{\frac{-2}{(2x+1)^2}} \end{aligned}$$

$$5) f(x) = \sqrt{x+3}$$

$$\lim_{h \rightarrow 0} \frac{\sqrt{x+h+3} - \sqrt{x+3}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{(\sqrt{x+h+3} - \sqrt{x+3})(\sqrt{x+h+3} + \sqrt{x+3})}{h(\sqrt{x+h+3} + \sqrt{x+3})}$$

$$= \lim_{h \rightarrow 0} \frac{x+h+3 - (x+3)}{h(\sqrt{x+h+3} + \sqrt{x+3})} = \lim_{h \rightarrow 0} \frac{\cancel{x}+h+\cancel{3} - \cancel{x}-\cancel{3}}{h(\sqrt{x+h+3} + \sqrt{x+3})}$$

$$= \lim_{h \rightarrow 0} \frac{h}{h(\sqrt{x+h+3} + \sqrt{x+3})} = \lim_{h \rightarrow 0} \frac{1}{\sqrt{x+h+3} + \sqrt{x+3}}$$

$$= \frac{1}{\sqrt{x+3} + \sqrt{x+3}} = \boxed{\frac{1}{2\sqrt{x+3}}}$$