$f(x)=x$

1. $\lim_{h\to 0} \frac{x+h-x}{h} = \lim_{h\to 0} \frac{h}{h} = \lim_{h\to 0} 1 = \boxed{1}$

$\lim_{x\to a} \frac{x-a}{x-a} = \lim_{x\to a} 1 = \boxed{1}$

2. $f(x) = 2x-3$

$\lim_{h\to 0} \frac{2(x+h)-3-(2x-3)}{h} = \lim_{h\to 0} \frac{2x+2h-3-2x+3}{h}$

$= \lim_{h\to 0} \frac{2h}{h} = \lim_{h\to 0} 2 = \boxed{2}$

$\lim_{x\to a} \frac{2x-3-(2a-3)}{x-a} = \lim_{x\to a} \frac{2x-2a}{x-a} = \lim_{x\to a} 2 = \boxed{2}$

3. $f(x) = x^2+1$

$\lim_{h\to 0} \frac{(x+h)^2+1-(x^2+1)}{h} = \lim_{h\to 0} \frac{x^2+2xh+h^2+1-x^2-1}{h}$

$= \lim_{h\to 0} \frac{2xh+h^2}{h} = \lim_{h\to 0} 2x+h = \boxed{2x}$

$\lim_{x\to a} \frac{x^2+1-(a^2+1)}{x-a} = \lim_{x\to a} \frac{x^2-a^2}{x-a} = \lim_{x\to a} \frac{(x+a)(x-a)}{(x-a)} = \lim_{x\to a} x+a = \boxed{2a}$

4. $f(x) = x^2+x+3$

$\lim_{h\to 0} \frac{(x+h)^2+x+h+3-(x^2+x+3)}{h} = \lim_{h\to 0} \frac{x^2+2xh+h^2+x+h+3-x^2-x-3}{h}$

$= \lim_{h\to 0} \frac{2xh+h^2+h}{h} = \lim_{h\to 0} 2x+h+1 = \boxed{2x+1}$

$$\lim_{x\to a} \frac{x^2+x+3-a^2-a-3}{x-a} = \lim_{x\to a} \frac{x^2-a^2}{x-a} + \lim_{x\to a} \frac{x-a}{x-a}$$

$$= \lim_{x\to a} \frac{(x+a)(x-a)}{x-a} + \lim_{x\to a} 1 = \lim_{x\to a} (x+a) + 1 = \boxed{2a+1}$$

5. $f(x) = 3x^3 - x - 1$

$$\lim_{h\to 0} \frac{3(x+h)^3 - (x+h) - 1 - 3x^3 + x + 1}{h}$$

$$= \lim_{h\to 0} \frac{3x^3 + 9x^2h + 9xh^2 + 3h^3 - x - h - 3x^3 + x}{h}$$

$$= \lim_{h\to 0} \frac{9x^2h + 9xh^2 + 3h^3 - h}{h} = \lim_{h\to 0} 9x^2 + 9xh + 3h^2 - 1 = \boxed{9x^2 - 1}$$

$$\lim_{x\to a} \frac{3x^3 - x - 1 - 3a^3 + a + 1}{x - a} = \lim_{x\to a} \frac{3x^3 - 3a^3}{x-a} - \lim_{x\to a} \frac{x-a}{x-a}$$

$$= \lim_{x\to a} \frac{3(x-a)(x^2 + ax + a^2)}{x-a} - \lim_{x\to a} 1 = \lim_{x\to a} 3(x^2+ax+a^2) - 1$$

$$= 3(a^2 + a^2 + a^2) - 1 = \boxed{9a^2 - 1}$$

6. $f(x) = \frac{1}{x}$

$$\lim_{h\to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \lim_{h\to 0} \frac{\frac{x}{x(x+h)} - \frac{x+h}{x(x+h)}}{h} = \lim_{h\to 0} \frac{x - x - h}{xh(x+h)}$$

$$= \lim_{h\to 0} \frac{-h}{xh(x+h)} = \lim_{h\to 0} \frac{-1}{x(x+h)} = \boxed{-\frac{1}{x^2}}$$

$$\lim_{x\to a} \frac{\frac{1}{x} - \frac{1}{a}}{x-a} = \lim_{x\to a} \frac{\frac{a}{ax} - \frac{x}{ax}}{x-a} = \lim_{x\to a} \frac{-(x-a)}{ax(x-a)}$$

$$= \lim_{x\to a} \frac{-1}{ax} = \boxed{-\frac{1}{a^2}}$$

7. $f(x) = \dfrac{2}{x^2}$

$$\lim_{h\to 0} \frac{\frac{2}{(x+h)^2} - \frac{2}{x^2}}{h} = \lim_{h\to 0} \frac{\frac{2x^2}{x^2(x+h)^2} - \frac{2(x+h)^2}{x^2(x+h)^2}}{h}$$

$$= \lim_{h\to 0} \frac{2x^2 - 2x^2 - 4xh - 2h^2}{x^2 h (x+h)^2} = \lim_{h\to 0} \frac{-4xh - 2h^2}{x^2 h (x+h)^2}$$

$$= \lim_{h\to 0} \frac{-4x - 2h}{x^2(x+h)^2} = \frac{-4x}{x^4} = \boxed{-\frac{4}{x^3}}$$

$$\lim_{x\to a} \frac{\frac{2}{x^2} - \frac{2}{a^2}}{x-a} = \lim_{x\to a} \frac{\frac{2a^2 - 2x^2}{x^2a^2}}{x-a} = \lim_{x\to a} \frac{2(a^2-x^2)}{x^2a^2(x-a)}$$

$$= \lim_{x\to a} \frac{2(a-x)(a+x)}{x^2a^2(x-a)} = \lim_{x\to a} \frac{-2(x-a)(a+x)}{x^2a^2(x-a)}$$

$$= \lim_{x\to a} \frac{-2(a+x)}{x^2a^2} = \frac{-2(a+a)}{a^4} = \frac{-4a}{a^4} = \boxed{-\frac{4}{a^3}}$$

8. $f(x) = \dfrac{2}{x+3}$

$$\lim_{h\to 0} \frac{\frac{2}{x+h+3} - \frac{2}{x+3}}{h} = \lim_{h\to 0} \frac{\frac{2(x+3)}{(x+3)(x+h+3)} - \frac{2(x+h+3)}{(x+3)(x+h+3)}}{h}$$

$$= \lim_{h\to 0} \frac{2x+6 - 2x - 2h - 6}{(x+3)(x+h+3)h} = \lim_{h\to 0} \frac{-2h}{(x+3)(x+h+3)h}$$

$$= \lim_{h\to 0} \frac{-2}{(x+3)(x+h+3)} = \boxed{-\frac{2}{(x+3)^2}}$$

$$\lim_{x\to a} \frac{\frac{2}{x+3} - \frac{2}{a+3}}{x-a} = \lim_{x\to a} \frac{\frac{2(a+3)}{(x+3)(a+3)} - \frac{2(x+3)}{(x+3)(a+3)}}{x-a}$$

$$= \lim_{x\to a} \frac{2a+6-2x-6}{(x+3)(a+3)(x-a)} = \lim_{x\to a} \frac{2(a-x)}{-(x+3)(a+3)(a-x)}$$

$$= \lim_{x\to a} \frac{-2}{(x+3)(a+3)} = \boxed{-\frac{2}{(a+3)^2}}$$

9. $f(x) = \sqrt{x}$

$$\lim_{h\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h} = \lim_{h\to 0} \frac{(\sqrt{x+h}-\sqrt{x})(\sqrt{x+h}+\sqrt{x})}{h(\sqrt{x+h}+\sqrt{x})}$$

$$= \lim_{h\to 0} \frac{x+h-x}{h(\sqrt{x+h}+\sqrt{x})} = \lim_{h\to 0} \frac{h}{h(\sqrt{x+h}+\sqrt{x})} = \lim_{h\to 0} \frac{1}{\sqrt{x+h}+\sqrt{x}} = \boxed{\frac{1}{2\sqrt{x}}}$$

$$\lim_{x\to a} \frac{\sqrt{x}-\sqrt{a}}{x-a} = \lim_{x\to a} \frac{(\sqrt{x}-\sqrt{a})(\sqrt{x}+\sqrt{a})}{(x-a)(\sqrt{x}+\sqrt{a})}$$

$$= \lim_{x\to a} \frac{(x-a)}{(x-a)(\sqrt{x}+\sqrt{a})} = \lim_{x\to a} \frac{1}{\sqrt{x}+\sqrt{a}} = \boxed{\frac{1}{2\sqrt{a}}}$$

10. $f(x) = \frac{1}{\sqrt{x}}$

$$\lim_{h\to 0} \frac{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}}{h} = \lim_{h\to 0} \frac{\frac{\sqrt{x}}{\sqrt{x}\sqrt{x+h}} - \frac{\sqrt{x+h}}{\sqrt{x}\sqrt{x+h}}}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{x}-\sqrt{x+h}}{\sqrt{x}\sqrt{x+h}\,(h)} = \lim_{h\to 0} \frac{(\sqrt{x}-\sqrt{x+h})(\sqrt{x}+\sqrt{x+h})}{\sqrt{x}\sqrt{x+h}\,(h)(\sqrt{x}+\sqrt{x+h})}$$

$$= \lim_{h\to 0} \frac{x-(x+h)}{\sqrt{x}\sqrt{x+h}\,(h)(\sqrt{x}+\sqrt{x+h})} = \lim_{h\to 0} \frac{-h}{\sqrt{x}\sqrt{x+h}\,(h)(\sqrt{x}+\sqrt{x+h})}$$

$$= \lim_{x\to 0} \frac{-1}{\sqrt{x}\sqrt{x+h}(\sqrt{x}+\sqrt{x+h})} = \frac{-1}{\sqrt{x}\sqrt{x}(\sqrt{x}+\sqrt{x})} = \boxed{\frac{-1}{2x\sqrt{x}}}$$

$$\lim_{x\to a} \frac{\frac{1}{\sqrt{x}} - \frac{1}{\sqrt{a}}}{x-a} = \lim_{x\to a} \frac{\frac{\sqrt{a}}{\sqrt{a}\sqrt{x}} - \frac{\sqrt{x}}{\sqrt{a}\sqrt{x}}}{x-a}$$

$$= \lim_{x\to a} \frac{\sqrt{a}-\sqrt{x}}{\sqrt{a}\sqrt{x}(x-a)} = \lim_{x\to a} \frac{(\sqrt{a}-\sqrt{x})(\sqrt{a}+\sqrt{x})}{\sqrt{a}\sqrt{x}(x-a)(\sqrt{a}+\sqrt{x})}$$

$$= \lim_{x\to a} \frac{a-x}{-\sqrt{a}\sqrt{x}(a-x)(\sqrt{a}+\sqrt{x})} = \lim_{x\to a} -\frac{1}{\sqrt{a}\sqrt{x}(\sqrt{a}+\sqrt{x})}$$

$$= -\frac{1}{\sqrt{a}\sqrt{a}(\sqrt{a}+\sqrt{a})} = \boxed{-\frac{1}{2a\sqrt{a}}}$$