Extra Practice

Directions: Complete each problem and show all work. Make sure to clearly box or circle your answers.

1. Find each limit (if it exists):

a. $\lim_{x \to 3} (x^2 + 2)$

b. $\lim_{x \to 100} 7$

c. $\lim_{x \to 5} \frac{x+2}{x-4}$

d. $\lim_{x \to 2} \frac{x^2-4}{x-2}$

e. $\lim_{x \to 1} \frac{x^2-x}{2x^2+5x-7}$

f. $\lim_{z \to 4} \frac{z^2-16}{\sqrt{z}-2}$

g. $\lim_{x \to 25} \frac{\sqrt{x}-5}{x-25}$

h. $\lim_{h \to 0} \frac{(x+h)^2-x^2}{h}$

i. $\lim_{a \to -2} \frac{a-4}{a^2-2a-8}$

2. Find each limit, if it exists: (1) $\lim_{x \to a^-} f(x)$, (2) $\lim_{x \to a^+} f(x)$, (3) $\lim_{x \to a} f(x)$

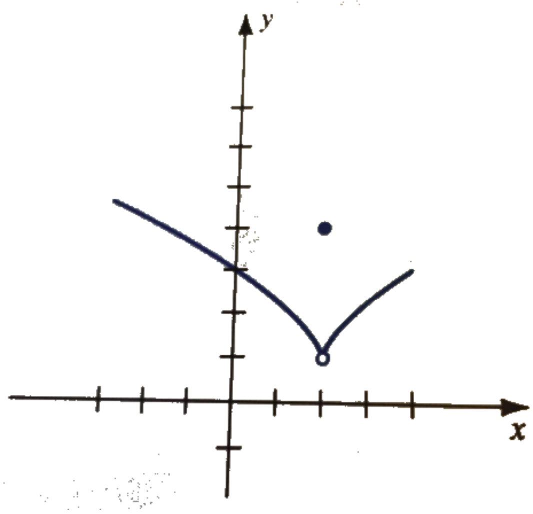
a. $f(x) = \frac{x+5}{|x+5|}; a = -5$

b. $f(x) = \sqrt{x-3} + 2; a = 3$

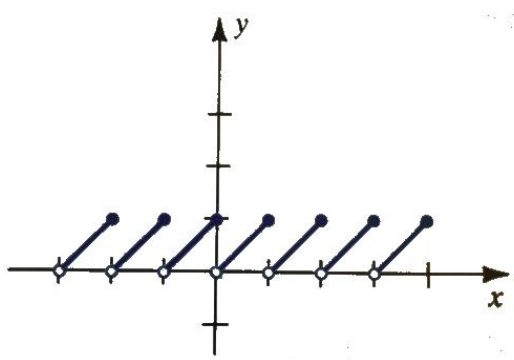
c. $f(x) = \frac{|x-1|}{x-1}; a = 1$

3. Refer to the graph to find each limit (if it exists): (1) $\lim_{x\to 2^-} f(x)$, (2) $\lim_{x\to 2^+} f(x)$, (3) $\lim_{x\to 2} f(x)$, (4) $\lim_{x\to 0^-} f(x)$, (5) $\lim_{x\to 0^+} f(x)$, (6) $\lim_{x\to 0} f(x)$

a.



b.



4. Use the Sandwich Theorem to verify $\lim_{x\to 0} x \sin\left(\frac{1}{x}\right) = 0$

5. Find each of the following limits using properties of limits and already known limit values:

a. $\lim\limits_{x \to 0} \frac{x}{\sin(2x)}$

b. $\lim\limits_{\theta \to 0} \frac{3\theta + \sin(\theta)}{\theta}$

c. $\lim\limits_{r \to 0} \frac{r\cos(r) - r^2}{r}$

d. $\lim\limits_{x \to 0} \frac{\tan(10x)}{\sin(3x)}$

e. $\lim\limits_{x \to 0} \frac{1 + 12x^2 - 2\cos(x) + \cos^2(x)}{3x^2}$

f. $\lim\limits_{z \to 0} \frac{\sin(7z)}{\sin(3z)}$

6. Express each of the following limits as $\infty, -\infty$, or DNE (Does Not Exist): (1) $\lim\limits_{x \to a^-} f(x)$, (2) $\lim\limits_{x \to a^+} f(x)$, (3) $\lim\limits_{x \to a} f(x)$

a. $f(x) = \frac{5}{x-4}; a = 4$

b. $f(x) = \frac{3x}{(x+8)^2}; a = -8$

c. $f(x) = \frac{1}{x(x-3)^2}; a = 3$

d. $f(x) = \frac{2x^2}{x^2 - x - 2}; a = -1$

7. Find each limit if it exists:

a. $\lim\limits_{x \to \infty} \frac{5x^2 - 3x + 1}{2x^2 + 4x - 7}$

b. $\lim\limits_{x \to \infty} \frac{2x^2 - x + 3}{x^3 + 1}$

c. $\lim\limits_{x \to -\infty} \frac{2 - x^2}{x + 3}$

d. $\lim\limits_{x \to -\infty} \frac{2x^2 - 3}{4x^3 + 5x}$

e. $\lim\limits_{x \to \infty} \sqrt[3]{\frac{8 + x^2}{x(x+1)}}$

f. $\lim\limits_{x \to -\infty} \frac{4x-3}{\sqrt{x^2+1}}$

g. $\lim\limits_{x \to \infty} \sin(x)$

h. $\lim\limits_{x \to \infty} \frac{x^2}{e^{2x}}$

i. $\lim\limits_{x \to \infty} (e^{8x} - 2e^{3x} + 5e^{2x} + 2e^{-2x})$

j. $\lim\limits_{x \to \infty} \ln(7x^2 - 2x + 5)$

k. $\lim\limits_{x \to -\infty} \ln \frac{1}{(x^2-x)}$

8. Find the vertical and horizontal asymptotes for each function. Justify your answer using limits:

a. $f(x) = \frac{1}{x^2-4}$

b. $f(x) = \frac{2x^2}{x+1}$

9. Verify the Intermediate Value Theorem for f on the stated interval $[a, b]$ by showing that if $f(a) \leq w \leq f(b)$, then $f(c) = w$ for some c in $[a, b]$.

a. $f(x) = x^3 + 1$ for $[-1,2]$

b. $f(x) = 2x - x^2$ for $[-2, -1]$

10. Prove that $f(x) = 3x^4 + 2x^3 - 63x^2 - 102x - 40$ has a solution between 0 and 6.

11. Determine A such that each function is continuous on $(-\infty, \infty)$:

a. $f(x) = \begin{cases} \frac{x^2-4}{x-2} & \text{if } x \neq 2 \\ A & \text{if } x = 2 \end{cases}$

b. $g(x) = \begin{cases} \frac{\sin(4x)}{x} & \text{if } x \neq 0 \\ A & \text{if } x = 0 \end{cases}$