Continuity and IVT

Directions: Do what the problems say to do.

1. Verify the Intermediate Value Theorem for f on the stated interval $[a, b]$ by showing that if $f(a) \leq w \leq f(b)$, then $f(c) = w$ for some c in (a, b).

a. $f(x) = x^3 + 1$ for $[-1,2]$

b. $f(x) = 2x - x^2$ for $[-2, -1]$

2. Prove that $f(x) = 3x^4 + 2x^3 - 63x^2 - 102x - 40$ has a solution between 0 and 6.

3. Determine A such that each function is continuous on $(-\infty, \infty)$:

a. $f(x) = \begin{cases} \frac{x^2-4}{x-2} & \text{if } x \neq 2 \\ A & \text{if } x = 2 \end{cases}$

b. $g(x) = \begin{cases} \frac{\sin(4x)}{x} & \text{if } x \neq 0 \\ A & \text{if } x = 0 \end{cases}$

4. Let f be defined as follows:

$$f(x) = \begin{cases} 2x + 1, & x \leq -2 \\ ax^2 + b, & -2 < x < 1 \\ \ln x, & x \geq 1 \end{cases}$$

Find values of a and b such that the function is continuous. Use the definition of continuity to justify your answer.