

$$1. a) \lim_{x \rightarrow 3} x^2 + 2 = 3^2 + 2 = \boxed{11}$$

$$b) \lim_{x \rightarrow 100} 7 = \boxed{7}$$

$$c) \lim_{x \rightarrow 5} \frac{x+2}{x-4} = \frac{7}{1} = \boxed{7}$$

$$d) \lim_{x \rightarrow 2} \frac{x^2-4}{x-2} = \lim_{x \rightarrow 2} \frac{(x+2)(x-2)}{x-2} = \lim_{x \rightarrow 2} x+2 = \boxed{4}$$

$$e) \lim_{x \rightarrow 1} \frac{x^2-x}{2x^2+5x-7} = \lim_{x \rightarrow 1} \frac{x(x-1)}{(x-1)(2x+7)} = \lim_{x \rightarrow 1} \frac{x}{2x+7} = \boxed{\frac{1}{9}}$$

$$f) \lim_{z \rightarrow 4} \frac{z^2-16}{\sqrt{z}-2} = \lim_{z \rightarrow 4} \frac{(z+4)(z-4)}{\sqrt{z}-2} = \lim_{z \rightarrow 4} \frac{(z+4)(\sqrt{z}+2)(\sqrt{z}-2)}{\sqrt{z}-2}$$

$$= \lim_{z \rightarrow 4} (z+4)(\sqrt{z}+2) = (8)(4) = \boxed{32}$$

$$g) \lim_{x \rightarrow 25} \frac{\sqrt{x}-5}{x-25} = \lim_{x \rightarrow 25} \frac{\sqrt{x}-5}{(\sqrt{x}+5)(\sqrt{x}-5)} = \lim_{x \rightarrow 25} \frac{1}{\sqrt{x}+5} = \boxed{\frac{1}{10}}$$

$$h) \lim_{h \rightarrow 0} \frac{(x+h)^2 - x^2}{h} = \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - x^2}{h} = \lim_{h \rightarrow 0} \frac{2xh + h^2}{h}$$

$$= \lim_{h \rightarrow 0} 2x + h = 2x + 0 = \boxed{2x}$$

$$i) \lim_{a \rightarrow -2} \frac{a-4}{a^2-2a-8} = \frac{-6}{4+4-8} = \frac{-6}{0} \quad \boxed{\text{DNE}}$$

$$2. a) 1) \lim_{x \rightarrow -5^-} \frac{x+5}{|x+5|} \quad \text{if } x \rightarrow -5^-, \quad x+5 < 0 \quad \text{so } = \lim_{x \rightarrow -5^-} \frac{x+5}{-(x+5)} = \lim_{x \rightarrow -5^-} -1 = \boxed{-1}$$

$$2) \lim_{x \rightarrow -5^+} \frac{x+5}{|x+5|} \quad \text{if } x \rightarrow -5^+, \quad x+5 > 0 \quad \text{so } = \lim_{x \rightarrow -5^+} \frac{x+5}{x+5} = \lim_{x \rightarrow -5^+} 1 = \boxed{1}$$

$$3) \lim_{x \rightarrow -5} \frac{x+5}{|x+5|} \quad \text{DNE} \quad \text{b/c left and right limits do not equal each other}$$

2. b) 1) $\lim_{x \rightarrow 3^-} \sqrt{x-3} + 2$ DNE because $\sqrt{x-3} + 2$ is undefined for $x < 3$

2) $\lim_{x \rightarrow 3^+} \sqrt{x-3} + 2 = \sqrt{3-3} + 2 = \boxed{2}$

3) $\lim_{x \rightarrow 3} \sqrt{x-3} + 2$ DNE because the left and right limits are not equal

c) 1) $\lim_{x \rightarrow 1^-} \frac{|x-1|}{x-1}$ if $x < 1$, $|x-1| = -(x-1)$ so $\lim_{x \rightarrow 1^-} \frac{-(x-1)}{x-1} = \lim_{x \rightarrow 1^-} -1 = \boxed{-1}$

2) $\lim_{x \rightarrow 1^+} \frac{|x-1|}{x-1}$ if $x \geq 1$, $|x-1| = x-1$ so $\lim_{x \rightarrow 1^+} \frac{x-1}{x-1} = \lim_{x \rightarrow 1^+} 1 = \boxed{1}$

3) $\lim_{x \rightarrow 1} \frac{|x-1|}{x-1}$ DNE because left and right limits are not equal

3. a) 1) 1 2) 1 3) 1 4) 3 5) 3 6) 3

b) 1) 1 2) 0 3) DNE 4) 1 5) 0 6) DNE

4. $\lim_{x \rightarrow 0} x \sin \frac{1}{x}$

$-1 \leq \sin \frac{1}{x} \leq 1$

if $x > 0$

$-x \leq x \sin \frac{1}{x} \leq x$

$\lim_{x \rightarrow 0^+} -x = 0 = \lim_{x \rightarrow 0^+} x$

so $\lim_{x \rightarrow 0^+} x \sin \frac{1}{x} = 0$

if $x < 0$

$-x \geq x \sin \frac{1}{x} \geq x$

$\lim_{x \rightarrow 0^-} -x = 0 = \lim_{x \rightarrow 0^-} x$

so $\lim_{x \rightarrow 0^-} x \sin \frac{1}{x} = 0$

so $\boxed{\lim_{x \rightarrow 0} x \sin \frac{1}{x} = 0}$

$$5. a) \lim_{x \rightarrow 0} \frac{x}{\sin 2x} = \lim_{x \rightarrow 0} \frac{2x}{2 \sin 2x} = \frac{1}{2} \lim_{x \rightarrow 0} \frac{2x}{\sin 2x} = \left(\frac{1}{2}\right)(1) = \boxed{\frac{1}{2}}$$

$$b) \lim_{\theta \rightarrow 0} \frac{3\theta + \sin \theta}{\theta} = \lim_{\theta \rightarrow 0} \frac{3\theta}{\theta} + \lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = \left(\lim_{\theta \rightarrow 0} 3\right) + 1 = 3 + 1 = \boxed{4}$$

$$c) \lim_{r \rightarrow 0} \frac{r \cos r - r^2}{r} = \lim_{r \rightarrow 0} \cos r - r = \cos 0 - 0 = \boxed{1}$$

$$d) \lim_{x \rightarrow 0} \frac{\tan 10x}{\sin 3x} = \lim_{x \rightarrow 0} \frac{\tan 10x}{1} \cdot \frac{1}{\sin 3x} = \lim_{x \rightarrow 0} \frac{10 \tan 10x}{10x} \cdot \frac{3x}{3 \sin 3x} = \frac{10}{3} \left[\lim_{x \rightarrow 0} \frac{\tan 10x}{10x} \right] \left[\lim_{x \rightarrow 0} \frac{3x}{\sin 3x} \right] = \left(\frac{10}{3}\right)(1)(1) = \boxed{\frac{10}{3}}$$

$$e) \lim_{x \rightarrow 0} \frac{1 + 12x^2 - 2 \cos x + \cos^2 x}{3x^2} = \frac{1}{3} \left[\lim_{x \rightarrow 0} \frac{1 - 2 \cos x + \cos^2 x}{x^2} + \frac{12x^2}{x^2} \right] = \frac{1}{3} \left[\lim_{x \rightarrow 0} \left(\frac{1 - \cos x}{x} \right)^2 + \lim_{x \rightarrow 0} 12 \right] = \frac{1}{3} \left[\left(\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} \right)^2 + 12 \right] = \frac{1}{3} (0 + 12) = \boxed{4}$$

$$f) \lim_{z \rightarrow 0} \frac{\sin 7z}{\sin 3z} = \lim_{z \rightarrow 0} \frac{\sin 7z}{1} \cdot \frac{1}{\sin 3z} = \lim_{z \rightarrow 0} \frac{7 \sin 7z}{7z} \cdot \frac{3z}{3 \sin 3z} = \frac{7}{3} \left[\lim_{z \rightarrow 0} \frac{\sin 7z}{7z} \right] \left[\lim_{z \rightarrow 0} \frac{3z}{\sin 3z} \right] = \left(\frac{7}{3}\right)(1)(1) = \boxed{\frac{7}{3}}$$

$$6. a) 1) \lim_{x \rightarrow 4^-} \frac{5}{x-4} = \frac{5}{\text{small } -} = -\infty$$

$$2) \lim_{x \rightarrow 4^+} \frac{5}{x-4} = \frac{5}{\text{small } +} = \infty$$

$$3) \lim_{x \rightarrow 4} \frac{5}{x-4} \text{ DNE b/c } -\infty \neq \infty$$

$$b) 1) \lim_{x \rightarrow -8^-} \frac{3x}{(x+8)^2} = \frac{-24}{\text{small } +} = -\infty$$

$$2) \lim_{x \rightarrow -8^+} \frac{3x}{(x+8)^2} = \frac{-24}{\text{small } +} = -\infty$$

$$3) \lim_{x \rightarrow -8} \frac{3x}{(x+8)^2} = -\infty$$

$$c) 1) \lim_{x \rightarrow 3^-} \frac{1}{x(x-3)^2} = \frac{1}{3(\text{small } +)} = \infty$$

$$2) \lim_{x \rightarrow 3^+} \frac{1}{x(x-3)^2} = \frac{1}{3(\text{small } +)} = \infty$$

$$3) \lim_{x \rightarrow 3} \frac{1}{x(x-3)^2} = \infty$$

$$d) 1) \lim_{x \rightarrow -1^-} \frac{2x^2}{x^2-x-2} = \lim_{x \rightarrow -1^-} \frac{2x^2}{(x-2)(x+1)} = \frac{2}{(-3)(\text{small } -)} = \infty$$

$$2) \lim_{x \rightarrow -1^+} \frac{2x^2}{x^2-x-2} = \lim_{x \rightarrow -1^+} \frac{2x^2}{(x-2)(x+1)} = \frac{2}{(-3)(\text{small } +)} = -\infty$$

$$3) \lim_{x \rightarrow -1} \frac{2x^2}{x^2-x-2} \text{ DNE b/c } \infty \neq -\infty$$

$$7. a) \lim_{x \rightarrow \infty} \frac{5x^2 - 3x + 1}{2x^2 + 4x - 7} = \lim_{x \rightarrow \infty} \frac{\frac{5x^2 - 3x + 1}{x^2}}{\frac{2x^2 + 4x - 7}{x^2}} = \lim_{x \rightarrow \infty} \frac{5 - \frac{3}{x} + \frac{1}{x^2}}{2 + \frac{4}{x} - \frac{7}{x^2}}$$

$$= \frac{5 - 0 + 0}{2 + 0 - 0} = \boxed{\frac{5}{2}}$$

$$b) \lim_{x \rightarrow \infty} \frac{2x^2 - x + 3}{x^3 + 1} = \lim_{x \rightarrow \infty} \frac{\frac{2x^2 - x + 3}{x^3}}{\frac{x^3 + 1}{x^3}} = \lim_{x \rightarrow \infty} \frac{\frac{2}{x} - \frac{1}{x^2} + \frac{3}{x^3}}{1 + \frac{1}{x^3}}$$

$$= \frac{0 - 0 + 0}{1 + 0} = \boxed{0}$$

$$c) \lim_{x \rightarrow -\infty} \frac{2 - x^2}{x + 3} = \lim_{x \rightarrow -\infty} \frac{\frac{2 - x^2}{x^2}}{\frac{x + 3}{x^2}} = \lim_{x \rightarrow -\infty} \frac{\frac{2}{x^2} - 1}{\frac{1}{x} + \frac{3}{x^2}}$$

$$= \frac{0 - 1}{0 + 0} = \frac{-1}{0} \quad \boxed{\text{DNE}}$$

$$d) \lim_{x \rightarrow -\infty} \frac{2x^2 - 3}{4x^3 + 5x} = \lim_{x \rightarrow -\infty} \frac{\frac{2x^2 - 3}{x^3}}{\frac{4x^3 + 5x}{x^3}} = \lim_{x \rightarrow -\infty} \frac{\frac{2}{x} - \frac{3}{x^3}}{4 + \frac{5}{x^2}}$$

$$= \frac{0 - 0}{4 + 0} = \boxed{0}$$

$$e) \lim_{x \rightarrow \infty} \sqrt[3]{\frac{8 + x^2}{x(x+1)}} = \sqrt[3]{\lim_{x \rightarrow \infty} \frac{\frac{8 + x^2}{x^2}}{\frac{x^2 + x}{x^2}}} = \sqrt[3]{\lim_{x \rightarrow \infty} \frac{\frac{8}{x^2} + 1}{1 + \frac{1}{x}}} = \sqrt[3]{\frac{0 + 1}{1 + 0}} = \boxed{1}$$

$$f) \lim_{x \rightarrow -\infty} \frac{4x - 3}{\sqrt{x^2 + 1}} = \lim_{x \rightarrow -\infty} \frac{\frac{4x - 3}{|x|}}{\sqrt{\frac{x^2 + 1}{x^2}}} = \lim_{x \rightarrow -\infty} \frac{\frac{4x - 3}{-x}}{\sqrt{1 + \frac{1}{x^2}}} = \lim_{x \rightarrow -\infty} \frac{-4 + \frac{3}{x}}{\sqrt{1 + \frac{1}{x^2}}}$$

b/c $|x| = -x$ if $x < 0$

$$= \frac{-4 + 0}{\sqrt{1 + 0}} = \boxed{-4}$$

g) $\lim_{x \rightarrow \infty} \sin x$ DNE b/c it oscillates between -1 and 1

h) $\lim_{x \rightarrow \infty} \frac{x^2}{e^{2x}} = \frac{\infty}{\infty \text{ faster}} = \boxed{0}$

i) $\lim_{x \rightarrow \infty} e^{8x} - 2e^{3x} + 5e^{2x} + 2e^{-2x} = \infty - \infty + \infty + 0$
so, indeterminate

$= \lim_{x \rightarrow \infty} e^{8x} (1 - 2e^{-5x} + 5e^{-6x} + 2e^{-10x}) = \infty (1 - 0 + 0 + 0)$
 $= \boxed{\infty}$

j) $\lim_{x \rightarrow \infty} \ln(7x^2 - 2x + 5)$ if $u = 7x^2 - 2x + 5$
 $x \rightarrow \infty, u \rightarrow \infty$

$= \lim_{u \rightarrow \infty} \ln u = \boxed{\infty}$

k) $\lim_{x \rightarrow -\infty} \ln \frac{1}{x^2 - x}$ if $u = \frac{1}{x^2 - x}$
 $x \rightarrow -\infty, u \rightarrow 0^+$

$= \lim_{u \rightarrow 0^+} \ln u = \boxed{-\infty}$

8. a) $f(x) = \frac{1}{x^2 - 4} = \frac{1}{(x+2)(x-2)}$ $x \neq -2$ $x \neq 2$

VA: $\left[\begin{array}{l} \lim_{x \rightarrow -2^-} f(x) = \frac{1}{(\text{small-})(-4)} = \infty \\ \lim_{x \rightarrow -2^+} f(x) = \frac{1}{(\text{small+})(-4)} = -\infty \\ \lim_{x \rightarrow 2^-} f(x) = \frac{1}{4(\text{small-})} = -\infty \\ \lim_{x \rightarrow 2^+} f(x) = \frac{1}{4(\text{small+})} = \infty \end{array} \right.$

HA: $\left[\begin{array}{l} \lim_{x \rightarrow \infty} \frac{1}{x^2 - 4} = \lim_{x \rightarrow \infty} \frac{\frac{1}{x^2}}{\frac{x^2 - 4}{x^2}} \\ = \lim_{x \rightarrow \infty} \frac{\frac{1}{x^2}}{1 - \frac{4}{x^2}} = \frac{0}{1 - 0} = 0 \\ \lim_{x \rightarrow -\infty} \frac{1}{x^2 - 4} = \lim_{x \rightarrow -\infty} \frac{\frac{1}{x^2}}{\frac{x^2 - 4}{x^2}} \\ = \lim_{x \rightarrow -\infty} \frac{\frac{1}{x^2}}{1 - \frac{4}{x^2}} = \frac{0}{1 - 0} = 0 \\ y = 0 \text{ in both directions} \end{array} \right.$

$$b) \lim_{x \rightarrow -1^-} \frac{2x^2}{x+1} = \frac{2}{\text{small}^-} = -\infty$$

$$f(x) = \frac{2x^2}{x+1} \quad x \neq -1$$

$$VA: \lim_{x \rightarrow -1^+} \frac{2x^2}{x+1} = \frac{2}{\text{small}^+} = \infty$$

$$HA: \lim_{x \rightarrow \infty} \frac{2x^2}{x+1} = \lim_{x \rightarrow \infty} \frac{\frac{2x^2}{x^2}}{\frac{x+1}{x^2}} = \lim_{x \rightarrow \infty} \frac{2}{\frac{1}{x} + \frac{1}{x^2}} = \frac{2}{0+0} \text{ DNE}$$

$$\lim_{x \rightarrow -\infty} \frac{2x^2}{x+1} = \lim_{x \rightarrow -\infty} \frac{\frac{2x^2}{x^2}}{\frac{x+1}{x^2}} = \lim_{x \rightarrow -\infty} \frac{2}{\frac{1}{x} + \frac{1}{x^2}} = \frac{2}{0+0} \text{ DNE}$$

no HA

$$9. a) f(x) = x^3 + 1 \text{ on } [-1, 2]$$

$f(x)$ continuous for all real numbers b/c it is a polynomial

$$f(-1) = 0 \quad f(2) = 9$$

so by IVT, for all w on $(0, 9)$ there is a c on $(-1, 2)$ such that $f(c) = w$

$$0 < w < 9$$

$$0 < c^3 + 1 < 9$$

$$-1 < c^3 < 8$$

$$-1 < c < 2 \quad \checkmark$$

$$b) f(x) = 2x - x^2 \text{ for } [-2, -1]$$

$f(x)$ continuous for all real numbers b/c it is a polynomial

$$f(-2) = -4 - 4 = -8 \quad f(-1) = -2 - 1 = -3$$

so by IVT for every w on $(-8, -3)$ There exists c on $(-2, -1)$ such that $f(c) = w$

$$-8 < w < -3$$

$$-8 < 2c - c^2 < -3$$

$$8 > c^2 - 2c > 3$$

$$9 > c^2 - 2c + 1 > 4$$

$$9 > (c-1)^2 > 4$$

$$-3 < c-1 < -2$$

$$-2 < c < -1 \quad \checkmark$$

$$10. f(x) = 3x^4 + 2x^3 - 63x^2 - 102x - 40$$

$f(x)$ is continuous on all real numbers b/c it is a polynomial

$$f(0) = -40$$

$$f(6) = 1400$$

by IVT $f(x) = 0$ on $(0, 6)$

because $-40 < 0 < 1400$

$$11. a) \quad f(x) = \begin{cases} \frac{x^2-4}{x-2} & \text{if } x \neq 2 \\ A & \text{if } x = 2 \end{cases}$$

$$\lim_{x \rightarrow 2} f(x) = \lim_{x \rightarrow 2} \frac{x^2-4}{x-2} = \lim_{x \rightarrow 2} \frac{(x-2)(x+2)}{x-2}$$

$$= \lim_{x \rightarrow 2} x+2 = 2+2 = 4 \quad \text{so } \lim_{x \rightarrow 2} \text{ exists}$$

$$f(2) = A$$

f continuous if $\boxed{A = 4}$

$$b) \quad g(x) = \begin{cases} \frac{\sin 4x}{x} & \text{if } x \neq 0 \\ A & \text{if } x = 0 \end{cases}$$

$$\lim_{x \rightarrow 0} g(x) = \lim_{x \rightarrow 0} \frac{\sin 4x}{x} = \lim_{x \rightarrow 0} \frac{4 \sin 4x}{4x}$$

$$= 4 \lim_{x \rightarrow 0} \frac{\sin 4x}{4x} = 4(1) = 4 \quad \text{so } \lim_{x \rightarrow 0} g(x) \text{ exists}$$

$$g(0) = A$$

$g(x)$ continuous if $\boxed{A = 4}$