

1. Find each of the following limits:

a. $\lim_{x \rightarrow 1} \frac{x-1}{x^{\frac{1}{6}}-1} = \frac{0}{0}$ $u = x^{\frac{1}{6}}$
 $x = u^6$

$x \rightarrow 1, u \rightarrow 1$

$$\lim_{u \rightarrow 1} \frac{u^6-1}{u-1} = \lim_{u \rightarrow 1} \frac{(u^3+1)(u^3-1)}{u-1}$$

$$= \lim_{u \rightarrow 1} \frac{(u^3+1)(u-1)(u^2+u+1)}{u-1} = \lim_{u \rightarrow 1} (u^3+1)(u^2+u+1)$$

$$= (2)(3) = \boxed{6}$$

b. $\lim_{x \rightarrow 0} \frac{x}{\sqrt[3]{x+8}-2} = \frac{0}{0}$

$u = \sqrt[3]{x+8}$

$x = u^3 - 8$

$x \rightarrow 0, u \rightarrow 2$

$$\lim_{u \rightarrow 2} \frac{u^3-8}{u-2} = \lim_{u \rightarrow 2} \frac{(u-2)(u^2+2u+4)}{u-2}$$

$$= \lim_{u \rightarrow 2} u^2+2u+4 = 4+4+4 = \boxed{12}$$

c. $\lim_{x \rightarrow 0} \frac{2x^2}{\sqrt[3]{(x+27)^2-6}\sqrt[3]{x+27}+9} = \frac{0}{0}$

$u = \sqrt[3]{x+27}$

$x = u^3 - 27$

$x \rightarrow 0, u \rightarrow 3$

$$\lim_{u \rightarrow 3} \frac{2(u^3-27)^2}{u^2-6u+9}$$

$$= \lim_{u \rightarrow 3} \frac{2(u-3)^2(u^2+3u+9)^2}{(u-3)^2} = \lim_{u \rightarrow 3} 2(u^2+3u+9)^2$$

$$= 2(9+9+9)^2 = \boxed{1458}$$

2. Verify the Intermediate Value Theorem for $f(x) = x^2 - 3x - 1$ on the interval $[1, 3]$.

$f(x)$ is continuous on $[1, 3]$ b/c it is a polynomial

$f(1) = -3$

$-3 < w < -1$

$f(c) = c^2 - 3c - 1 = w$

$f(3) = -1$

$-3 < c^2 - 3c - 1 < -1$

$-\frac{1}{2} < c - \frac{3}{2} < \frac{3}{2}$

$\frac{1}{4} < c^2 - 3c + \frac{9}{4} < \frac{9}{4}$

$\frac{1}{4} < (c - \frac{3}{2})^2 < \frac{9}{4}$

$1 < c < 3 \quad \checkmark$

3. Determine A such that the given function is continuous at $x = 1$.

$f(1) = A + 5$ ① $\checkmark$

$\lim_{x \rightarrow 1} f(x) = \lim_{x \rightarrow 1} \frac{x^3 - 3x^2 + 2}{x^2 - 1} = \frac{0}{0}$ $f(x) = \begin{cases} \frac{x^3 - 3x^2 + 2}{x^2 - 1}, & x \neq 1 \\ Ax + 5, & x = 1 \end{cases}$

$= \lim_{x \rightarrow 1} \frac{(x-1)(x^2-2x-2)}{(x+1)(x-1)}$

$= \lim_{x \rightarrow 1} \frac{x^2-2x-2}{x+1} = \frac{1^2-2(1)-2}{1+1} = -\frac{3}{2}$ ② $\checkmark$

③ $A + 5 = -\frac{3}{2}$

$A = -\frac{13}{2}$

$$\begin{array}{r} 1 \overline{) 1 \quad -3 \quad 0 \quad 2} \\ \underline{1 \quad -2 \quad -2} \\ 1 \quad -2 \quad -2 \quad 0 \end{array}$$