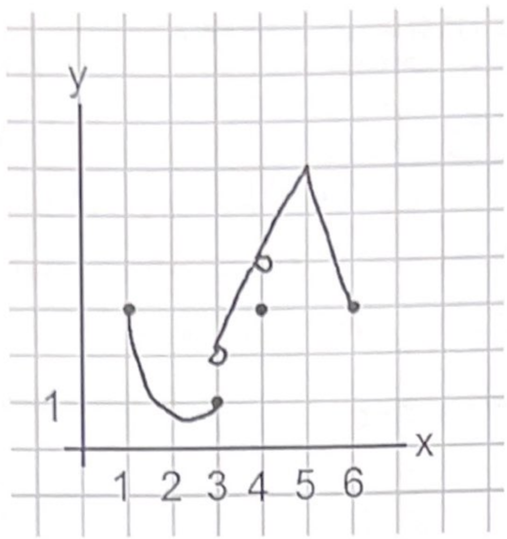


1. The graph of the function f is shown above. Which of the following statements is false?

a. $\lim_{x \to 1^+} f(x)$ exists   true $= 3$

b. $\lim_{x \to 2} f(x)$ exists   true $\approx .75$

c. $\lim_{x \to 3} f(x)$ exists   false $\lim_{x \to 3^-} f(x) = 1$ $\lim_{x \to 3^+} f(x) = 2$ so $\lim_{x \to 3} f(x)$ DNE

d. $\lim_{x \to 4} f(x)$ exists   true $= 4$

e. $\lim_{x \to 5} f(x)$ exists   true $= 6$

2. The line $y = 3$ is a horizontal asymptote to the graph of which of the following functions?

a. $f(x) = \frac{3x^2}{1-x^2}$ HA $y = -3$

b. $f(x) = \frac{\cos(3x)}{x}$ no HA

c. $f(x) = \frac{x}{3x-1}$ HA $y = \frac{1}{3}$

d. $f(x) = \frac{12x^2-1}{1+6x+4x^2}$ HA $= \frac{12}{4} = 3$

e. $f(x) = 3x$ no HA

3. Let f(x) be a function that is continuous on the closed interval [3,7] with f(3) = 5 and f(7) = 25. Which of the following is guaranteed by the Intermediate Value Theorem?

a. f(x) = 11 has at least one solution on the open interval (3,7) yes, because $5 < 11 < 25$

b. f(5) = 15 not guaranteed, could be true

c. f attains a minimum on the open interval (3,7) not guaranteed, could be true

d. f(x) = 5 has at least one solution on the open interval (3,7) not guaranteed, could be true

e. f(x) > 0 for every x in the open interval (3,7) not guaranteed, could be true

4. $\lim_{x \to -3} \frac{x^2+5x+6}{x^2-9}$ is

$$= \frac{0}{0} = \lim_{x \to -3} \frac{(x+2)(x+3)}{(x-3)(x+3)} = \lim_{x \to -3} \frac{x+2}{x-3}$$

$$= \frac{-3+2}{-3-3} = \frac{-1}{-6} = \frac{1}{6}$$

a. 0

b. 1

c. $\frac{1}{6}$

d. $-\frac{5}{6}$

e. non-existent

5. Let f be the function given by $f(x) = \frac{(x+2)(x-1)(x-3)}{(x+2)^2(x-2)}$. For which of the following values of x is the function not continuous.

$x+2 \neq 0 \quad x-2 \neq 0$

$x \neq -2 \quad x \neq 2$

a. 2 and -3 only

b. 2 only

c. 1 and 3 only

d. 2 and -2

e. 1, 3, and -2

6. For which of the following does $\lim_{x\to\infty} f(x) = 0$

I. $f(x) = \frac{e^x}{\ln x}$ $= \frac{\infty \text{ faster}}{\infty} = \infty$

II. $f(x) = \frac{x^{100}}{e^x}$ $= \frac{\infty}{\infty \text{ faster}} = 0$

III. $f(x) = \frac{\ln x}{x^{99}}$ $= \frac{\infty}{\infty \text{ faster}} = 0$

a. II and III

b. I only

c. I and II only

d. II only

e. I, II and III

7. $\lim_{x\to7^-} \frac{|x-7|}{x-7}$ is $= \frac{0}{0}$

as $x \to 7^-$, $x-7 < 0$ so $|x-7| = -(x-7)$

$$\lim_{x\to7^-} \frac{|x-7|}{x-7} = \lim_{x\to7^-} \frac{-(x-7)}{(x-7)} = \lim_{x\to7^-} -1 = -1$$

a. -7

b. -1

c. 1

d. 7

e. non-existent

8. Let f be the function defined by $f(x) = \frac{(3x+2)(1-2x)}{(3x+4)^2}$. Which of the following is its horizontal asymptote?

a. $y = 0$

b. $y = 1$

c. $y = \frac{1}{3}$

d. $y = -\frac{2}{3}$

e. $y = -\frac{2}{9}$

$$f(x) = \frac{-6x^2 - x + 2}{9x^2 + 24x + 16}$$

$$\lim_{x\to\infty} \frac{\frac{-6x^2-x+2}{x^2}}{\frac{9x^2+24x+16}{x^2}} = \lim_{x\to\infty} \frac{-6 - \frac{1}{x} + \frac{2}{x^2}}{9 - \frac{24}{x} + \frac{16}{x^2}}$$

$$= \frac{-6}{9} = -\frac{2}{3}$$

9. $\lim_{x\to\infty} \frac{x^5}{e^{5x}}$ $= \frac{\infty}{\infty \text{ faster}} = 0$

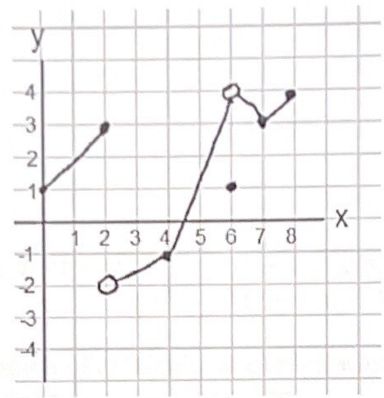
a. 0

b. $\frac{2}{25}$

c. $\frac{2}{5}$

d. 1

e. ∞



10. The graph of f(x) is shown above. Which of the following are true about f(x)?

I. $\lim_{x\to 6} f(x) = f(6)$ no, $\lim_{x\to 6} f(x) = 4$ but $f(6) = 1$

II. $\lim_{x\to 2^+} f(x) = f(2)$ no, $\lim_{x\to 2^+} f(x) = -2$ but $f(2) = 3$

III. $\lim_{x\to 6^-} f(x) = \lim_{x\to 6^+} f(x)$ yes both are 4

a. I only

b. III only

c. I and II only

d. I and III only

e. I, II, and III

11. Let f be a function defined by $f(x) = e^x(2x^2 - x - 3)$.

a. Find $\lim_{x\to\infty} f(x)$ $= e^{\infty}(\infty^2) = \boxed{\infty}$

b. Find $\lim_{x\to-\infty} f(x)$ $= e^{-\infty}(\infty^2) = \frac{\infty^2}{e^{\infty}} = \frac{\infty}{\infty \text{ faster}} = \boxed{0}$

12. Let g be defined as the function $g(x) = \sqrt{25 - x^2}$ if $-5 \le x \le -3$
$x + 7$ if $-3 < x \le 5$

Is g continuous at x = -3? Use the definition of continuity to explain your answer.

① ✓ $g(-3) = \sqrt{25-(-3)^2} = \sqrt{25-9} = 4$

② ✓ $\lim_{x\to -3^-} g(x) = \sqrt{25-(-3)^2} = 4$ $\lim_{x\to -3^+} g(x) = -3+7 = 4$

so $\lim_{x\to -3} g(x) = 4$

$\boxed{\text{yes}}$

③ ✓ $g(-3) = 4 = \lim_{x\to -3} g(x)$

13.

$$\lim_{x\to\infty} \frac{x^3 - 2x^2 + 3x - 4}{4x^3 - 3x^2 + 2x - 1} \overset{\frac{\infty}{\infty}}{=} \lim_{x\to\infty} \frac{\frac{x^3 - 2x^2 + 3x - 4}{x^3}}{\frac{4x^3 - 3x^2 + 2x - 1}{x^3}}$$

a. 4
b. 1
c. $\frac{1}{4}$
d. 0
e. −1

$$= \lim_{x\to\infty} \frac{1 - \frac{2}{x} + \frac{3}{x^2} - \frac{4}{x^3}}{4 - \frac{3}{x} + \frac{2}{x^2} - \frac{1}{x^3}} = \frac{1}{4}$$

14.

For $x \geq 0$, the horizontal line $y = 2$ is an asymptote for the graph of the function f. Which of the following statements must be true?

a. $f(0) = 2$ no, has nothing to do with asymptotes

b. $f(x) \neq 2$ for all $x \geq 0$ no, horizontal asymptotes can be crossed

c. $f(2)$ is undefined no, has nothing to do with asymptotes

d. $\lim_{x\to 2} f(x) = \infty$ no, this would indicate a vertical asymptote

e. $\lim_{x\to\infty} f(x) = 2$ yes, this is the definition of a horizontal asymptote

15.

Let f be the function defined by

$$f(x) = \begin{cases} \sqrt{x+1} & for\ 0 \leq x \leq 3 \\ 5 - x & for\ 3 < x \leq 5 \end{cases}$$

Is f continuous at $x = 3$? Explain why or why not.

① $f(3) = \sqrt{3+1} = 2$

② $\lim_{x\to 3^-} f(x) = \sqrt{3+1} = 2$

$\lim_{x\to 3^+} f(x) = 5-3 = 2$

so $\lim_{x\to 3} f(x) = 2$

③ $f(3) = 2 = \lim_{x\to 3} f(x)$, $\boxed{\text{yes}}$

16.

Let f be the function given by $f(x) = 2xe^{2x}$.

a. Find

$\lim_{x\to\infty} f(x) = (\infty)e^{\infty} = \boxed{\infty}$

b. Find

$\lim_{x\to-\infty} f(x) = \dfrac{-\infty}{e^{\infty}} = \dfrac{-\infty}{\infty \text{ faster}} = \boxed{0}$

17.

Let f be the function defined as follows

$f(x) = \begin{cases} |x-1| + 2 & for\ x < 1 \\ ax^2 + bx & for\ x \geq 1 \end{cases}$ where a and b are constants.

a. If $a = 2$ and $b = 3$, is f continuous for all x? Justify your answer.

b. Describe all values of a and b for which f is a continuous function.

a) $f(x) = \begin{cases} |x-1| + 2 & \text{for } x < 1 \\ 2x^2 + 3x & \text{for } x \geq 1 \end{cases}$

$f(x)$ cont. $(-\infty, 1) \cup (1, \infty)$ b/c absolute value and polynomial functions are continuous on $(-\infty, \infty)$

①

at $x = 1$, $f(1) = 2(1^2) + 3 = 5$

② $\lim_{x\to 1^-} f(x) = |1-1| + 2 = 2$

$\lim_{x\to 1^+} f(x) = 2(1^2) + 3(1) = 5$

so $\lim_{x\to 1} f(x)$ DNE $\boxed{\text{NO}}$

b) $\lim_{x\to 1^-} f(x)$ must $= \lim_{x\to 1^+} f(x)$

$2 = a(1^2) + b(1)$

$2 = a + b$

all values of a and b such that $a + b = 2$