1. $f(x) = \begin{cases} 3x-5 & \text{if } x \le 1 \\ ax^2+bx+1 & \text{if } 1 < x < 2 \\ 2x & \text{if } x \ge 2 \end{cases}$

f(x) is continuous on each separate interval because each piece is a polynomial

$f(1) = -2$

$\lim_{x \to 1^-} f(x) \stackrel{?}{=} \lim_{x \to 1^+} f(x)$

$-2 = a+b+1$

$a+b=-3$

$f(2) = 4$

$\lim_{x \to 2^-} f(x) \stackrel{?}{=} \lim_{x \to 2^+} f(x)$

$4a+2b+1 = 4$

$4a+2b=3$

$2a+2b=-6$

$2a = 9$

$a = \frac{9}{2}$

$\frac{9}{2} + b = -3$

$b = -\frac{15}{2}$

2. $f(x) = \begin{cases} ax^2+bx+1 & \text{if } x \le 1 \\ 2x+a+2b & \text{if } 1 < x < 2 \\ 3x-3 & \text{if } x \ge 2 \end{cases}$

f(x) is continuous on each separate interval because each piece is a polynomial

$f(1) = a+b+1$

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x)$

$a+b+1 = 2+a+2b$

$b=-1$

$f(2) = 3$

$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x)$

$4+a+2b = 3$

$4+a-2 = 3$

$a = 1$

3. $f(x) = \begin{cases} 2x+1 & \text{if } x \le -1 \\ ax^2+bx & \text{if } -1 < x < 1 \\ \ln x & \text{if } x \ge 1 \end{cases}$

f(x) is continuous on each piece because polynomial and logarithmic functions are continuous on their domains

$f(-1) = -1$

$\lim_{x \to -1^-} f(x) = \lim_{x \to -1^+} f(x)$

$-1 = a - b$

$f(1) = 0$

$\lim_{x \to 1^-} f(x) = a+b \quad \lim_{x \to 1^+} f(x) = 0$

$a+b=0$

$$\begin{aligned} a-b &= -1 \\ a+b &= 0 \\ \hline 2a &= -1 \end{aligned}$$

$a = -\frac{1}{2}$

$-\frac{1}{2} + b = 0$

$b = \frac{1}{2}$

4. $f(x) = \begin{cases} x^2+bx+4 & \text{if } x \le 0 \\ ax+2b+2 & \text{if } 0 < x < 3 \\ bx-2a+5 & \text{if } x \ge 3 \end{cases}$

f(x) is continuous on each piece because polynomials are continuous for all real numbers

$f(0) = 4$

$\lim_{x \to 0^-} f(x) = 4 \quad \lim_{x \to 0^+} f(x) = 2b+2$

$2b+2=4$

$b=1$

$\lim_{x \to 3^-} f(x) = 3a+2b+2 \quad f(3) = 3b-2a+5$

$\lim_{x \to 3^+} f(x) = 3b-2a+5$

$3a+4 = 8-2a$

$a = \frac{4}{5}$

5. $f(x) = \begin{cases} 2ax^2+x & \text{if } x \le 1 \\ 3x+a+2b & \text{if } 1 < x < 2 \\ 4x-a-b & \text{if } x \ge 2 \end{cases}$

f(x) is continuous on each piece because polynomials are continuous for all real numbers

$f(1) = 2a+1$

$\lim_{x \to 1^-} f(x) = 2a+1$

$\lim_{x \to 1^+} f(x) = 3+a+2b$

$2a+1 = 3+a+2b$

$a = 2+2b$

$a = 2+2\left(-\frac{2}{7}\right)$

$a = \frac{10}{7}$

$f(2) = 8-a-b$

$\lim_{x \to 2^-} f(x) = 6+a+2b$

$\lim_{x \to 2^+} f(x) = 8-a-b$

$6+a+2b = 8-a-b$

$2a+3b=2$

$2(2+2b)+3b=2$

$4+7b=2$

$b = -\frac{2}{7}$