1.a) $f(x) = x^3+1$ on $[-1,2]$

$f(x)$ is continuous because it is a polynomial

$f(-1) = 0$

$f(2) = 9$

so, by IVT forevery $w$ on $(0,9)$,

There exists a $c$ on $(-1,2)$

such that $f(c) = w$

$f(c) = c^3+1$

$0 < w < 9$

$0 < c^3+1 < 9$

$-1 < c^3 < 8$

$-1 < c < 2$ ✓

b) $f(x) = 2x - x^2$ on $[-2,-1]$

$f(x)$ is continuous because it is a polynomial

$f(-2) = -8$

$f(-1) = -3$

so, by IVT, for every $w$ on $(-8,-3)$,

There exists a $c$ on $(-2,-1)$

such that $f(c) = w$

$f(c) = 2c - c^2$

$-8 < w < -3$

$-8 < 2c - c^2 < -3$

$8 > c^2 - 2c > 3$

$9 > c^2 - 2c + 1 > 4$

$9 > (c-1)^2 > 4$

$-3 < c-1 < -2$

$-2 < c < -1$ ✓

2. $f(x) = 3x^4 + 2x^3 - 63x^2 - 102x - 40$

looking for a root on $(0,6)$

f is continuous because it is a polynomial

$f(0) = -40$

$f(6) = 1400$

$-40 < 0 < 1400$, by IVT There exists a root on $(0,6)$ ✓

3. a) $f(x) = \begin{cases} \frac{x^2-4}{x-2} & \text{if } x \neq 2 \\ A & \text{if } x = 2 \end{cases}$

$x^2-4$ continuous on $(-\infty, 2) \cup (2, \infty)$

$f(2) = A$

$\lim_{x \to 2} f(x) = \lim_{x \to 2} \frac{x^2-4}{x-2}$

$= \lim_{x \to 2} \frac{(x+2)(x-2)}{(x-2)}$

$= \lim_{x \to 2} x+2 = 4$

$f(2) = A = \lim_{x \to 2} f(x) = 4$ $\boxed{A = 4}$

b) $f(x) = \begin{cases} \frac{\sin 4x}{x} & \text{if } x \neq 0 \\ A & \text{if } x = 0 \end{cases}$

$\frac{\sin 4x}{4}$ continuous on $(-\infty, 0) \cup (0, \infty)$

$f(0) = A$

$\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{\sin 4x}{x} = \lim_{x \to 0} \frac{4 \sin 4x}{4x}$

$= 4 \lim_{x \to 0} \frac{\sin 4x}{4x} = 4(1) = 4$

$f(0) = A = \lim_{x \to 0} f(x) = 4$ $\boxed{A = 4}$

4.

$$f(x) = \begin{cases} 2x+1 & \text{if } x \le -2 \\ ax^2+b & \text{if } -2 < x < 1 \\ \ln x & \text{if } x \ge 1 \end{cases}$$

$2x+1$ is continuous on $(-\infty, -2)$

$ax^2+b$ is continuous on $(-2, 1)$

$\ln x$ is continuous on $(1, \infty)$

$$\lim_{x \to -2^-} f(x) = \lim_{x \to -2^+} f(x)$$

$$\lim_{x \to -2^-} 2x+1 = \lim_{x \to -2^+} ax^2+b$$

$$2(-2)+1 = a(-2)^2+b$$

$$-3 = 4a+b$$

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x)$$

$$\lim_{x \to 1^-} ax^2+b = \lim_{x \to 1^+} \ln x$$

$$a(1^2)+b = \ln(1)$$

$$a+b = 0$$

$$a = -b$$

$$-3 = 4(-b)+b$$

$$-3 = -3b$$

$$\boxed{b=1}$$

$$\boxed{a=-1}$$