1. $f(x) = \dfrac{x^2-5x+6}{x^2-7x+10} = \dfrac{(x-2)(x-3)}{(x-2)(x-5)}$ $x=2$, $x=5$ are possibilities

$\lim_{x\to 2} f(x) = \lim_{x\to 2} \dfrac{x-3}{x-5} = \dfrac{-1}{-3} = \dfrac{1}{3}$ $f(2)$ DNE $x=2$ removable

$\lim_{x\to 5} f(x) = \dfrac{2}{0} \to \pm\infty$ $x=5$ infinite

2. $f(x) = \dfrac{1-\cos x}{x(3-x)}$ $x=0$, $x=3$ are possibilities

$\lim_{x\to 0} f(x) = \left[\lim_{x\to 0} \dfrac{1-\cos x}{x}\right]\left[\lim_{x\to 0} \dfrac{1}{3-x}\right] = 0\left(\dfrac{1}{3}\right) = 0$ $f(0)$ DNE

$x=0$ removable

$\lim_{x\to 3} f(x) = \dfrac{1-\cos(3)}{0} = \pm\infty$ $x=3$ infinite

3. $f(x) = \dfrac{(x+4)(x^2-x)}{x^2+8x} = \dfrac{x(x+4)(x-1)}{x(x+8)}$ $x=0$, $x=-8$ are possibilities

$\lim_{x\to 0} f(x) = \lim_{x\to 0} \dfrac{(x+4)(x-1)}{x+8} = \dfrac{(4)(-1)}{8} = -\dfrac{1}{2}$ $f(0)$ DNE

$x=0$ removable

$\lim_{x\to -8} f(x) = \dfrac{(-8)(-4)(-9)}{(-8)(0)} = \pm\infty$ $x=-8$ infinite

4. $f(x) = \begin{cases} x+3 & \text{if } x < -2 \\ x^2+1 & \text{if } -2 \le x \le 0 \\ x+1 & \text{if } x > 0 \end{cases}$

all pieces are continuous on each separate piece where they are defined

$\lim_{x \to -2^-} f(x) = -2+3 = 1$ $\lim_{x \to -2^+} f(x) = (-2)^2+1 = 5$ $1 \neq 5$

jump discontinuity at $x = -2$

$\lim_{x \to 0^-} f(x) = 0^2+1 = 1$ $\lim_{x \to 0^+} f(x) = 0+1 = 1$ $1 = 1$ no discontinuity

5. $f(x) = \begin{cases} \frac{1}{x} & \text{if } x < 0 \\ \ln x - 4 & \text{if } 0 < x < 1 \\ x^3 & \text{if } x \ge 1 \end{cases}$

all pieces are continuous on each separate piece where they are defined

$\lim_{x \to 0^-} f(x) = \frac{1}{\text{small} -} = -\infty$ $\lim_{x \to 0^+} f(x) = \ln(0^+) - 4 = -\infty$

infinite discontinuity at $x = 0$

$\lim_{x \to 1^-} f(x) = \ln 1 - 4 = -4$ $\lim_{x \to 1^+} f(x) = 1^3 = 1$ $-4 \neq 1$

jump discontinuity at $x = 1$