1. $\lim\limits_{x\to 4} \dfrac{3\sqrt[3]{(x+4)^2} - 6\sqrt[3]{x+4}}{\sqrt[3]{(x+4)^2} - 4} = \dfrac{12-12}{4-4} = \dfrac{0}{0}$

$u = \sqrt[3]{x+4}$ $\quad x = u^3 - 4$ $\quad$ as $x \to 4$, $u \to \sqrt[3]{4+4} = 2$

$\lim\limits_{u\to 2} \dfrac{3u^2 - 6u}{u^2 - 4} = \dfrac{0}{0}$ $\quad \lim\limits_{u\to 2} \dfrac{3u\cancel{(u-2)}}{(u+2)\cancel{(u-2)}} = \lim\limits_{u\to 2} \dfrac{3u}{u+2} = \dfrac{6}{4} = \boxed{\dfrac{3}{2}}$

2. $\lim\limits_{x\to 0} \dfrac{x}{\sqrt[3]{(x+8)^2} - \sqrt[3]{x+8} - 2} = \dfrac{0}{4-2-2} = \dfrac{0}{0}$ $\quad u = \sqrt[3]{x+8}$ $\quad x = u^3 - 8$

$x \to 0$, $u \to \sqrt[3]{0+8} = 2$

$\lim\limits_{u\to 2} \dfrac{u^3 - 8}{u^2 - u - 2} = \dfrac{0}{0} = \lim\limits_{u\to 2} \dfrac{\cancel{(u-2)}(u^2+2u+4)}{\cancel{(u-2)}(u+1)}$

$= \lim\limits_{u\to 2} \dfrac{u^2+2u+4}{u+1} = \dfrac{2^2+2(2)+4}{2+1} = \dfrac{12}{3} = \boxed{4}$

3. $\lim\limits_{x\to\infty} \ln\left(\dfrac{-3}{x^2-4}\right)$ $\quad u = -\dfrac{3}{x^2-4}$ $\quad x \to \infty$, $u \to \dfrac{-3}{\infty} = 0^-$

$= \lim\limits_{u\to 0^-} \ln u$ $\boxed{\text{DNE}}$ b/c $\ln u$ is not defined for negative values

4. $\lim\limits_{x\to -\infty} \ln(2+e^x) = \ln(2+e^{-\infty}) = \ln\left(2 + \dfrac{1}{e^\infty}\right) = \ln(2+0) = \boxed{\ln 2}$

5. $\lim\limits_{x\to -\infty} \ln(e^x + x^2) = \ln(e^{-\infty} + (-\infty)^2) = \ln\left(\dfrac{1}{e^\infty} + \infty\right) = \ln(0+\infty)$
$= \boxed{\infty}$

6. $\lim\limits_{x\to -\infty} e^{(4x-3)} = e^{-\infty} = \dfrac{1}{e^\infty} = \boxed{0}$

7. $\lim\limits_{x\to\infty} 2^{\left(\frac{1}{x}+x\right)} = 2^{\frac{1}{\infty}+\infty} = 2^{0+\infty} = 2^\infty = \boxed{\infty}$

8. $\lim_{x \to -\infty} \frac{\sqrt{2x^2+5}}{4x-3} = \lim_{x \to -\infty} \frac{\sqrt{\frac{2x^2+5}{x^2}}}{\frac{4x-3}{|x|}} = \lim_{x \to -\infty} \frac{\sqrt{2+\frac{5}{x^2}}}{\frac{4x-3}{-x}}$

$|x| = -x$ if $x < 0$

$= \lim_{x \to -\infty} \frac{\sqrt{2+\frac{5}{x^2}}}{-4+\frac{3}{x}} = \boxed{-\frac{\sqrt{2}}{4}}$

9. $\lim_{x \to \infty} \frac{\sqrt{9x^2+x+2}}{2x-1} = \lim_{x \to \infty} \frac{|3x|}{2x}$ since $x \to \infty$, $x > 0$ and $3x > 0$

so $= \lim_{x \to \infty} \frac{3x}{2x} = \lim_{x \to \infty} \frac{3}{2} = \boxed{\frac{3}{2}}$

10. $\lim_{x \to -\infty} \sqrt{\frac{25x^2-4}{x^2+1}}$ $u = \frac{25x^2-4}{x^2+1}$ as $x \to -\infty$, $u \to \frac{25}{1} = 25$

$\lim_{u \to 25} \sqrt{u} = \sqrt{25} = \boxed{5}$

11. $\lim_{x \to -\infty} \frac{3x+8}{\sqrt{36x^2-5}} = \lim_{x \to -\infty} \frac{3x}{|6x|}$ since $x \to -\infty$, $x < 0$ $6x < 0$

so $\lim_{x \to -\infty} \frac{3x}{-6x} = \lim_{x \to -\infty} \frac{3}{-6} = \boxed{-\frac{1}{2}}$

12. $\lim_{x \to \infty} \frac{x^{999}}{2^x} = \frac{\infty}{\infty \text{ faster}} = \boxed{0}$

13. $\lim_{x \to \infty} \frac{\sqrt[4]{x}}{\log_5 x} = \frac{\infty \text{ faster}}{\infty} = \boxed{\infty}$

14. $\lim_{x \to \infty} \frac{x^5}{\sqrt{x}} = \frac{\infty \text{ faster}}{\infty} = \boxed{\infty}$

15. $\lim_{x \to \infty} \frac{\ln x}{e^x} = \frac{\infty}{\infty \text{ faster}} = \boxed{0}$