1. $\lim\limits_{x \to 4} 2x-5 = 3$

Proof: Given $\epsilon > 0$, choose $\delta = \frac{\epsilon}{2}$

suppose $0 < |x-4| < \delta$

Then $|2x-5-3|$

$= |2x-8|$

$= |2| \cdot |x-4|$

$= 2|x-4|$

$< 2\delta = 2\left(\frac{\epsilon}{2}\right) = \epsilon$

2. $\lim\limits_{x \to -1} 3x+7 = 4$

Proof: Given $\epsilon > 0$, choose $\delta = \frac{\epsilon}{3}$

Suppose $0 < |x-(-1)| < \delta \Rightarrow 0 < |x+1| < \delta$

Then $|3x+7-4|$

$= |3x+3|$

$= 3|x+1|$

$< 3\delta = 3\left(\frac{\epsilon}{3}\right) = \epsilon$

3. $\lim\limits_{x \to -3} -2x-8 = -2$

Proof: Given $\epsilon > 0$, choose $\delta = \frac{\epsilon}{2}$

Suppose $0 < |x-(-3)| < \delta \Rightarrow 0 < |x+3| < \delta$

Then $|-2x-8-(-2)$

$= |-2x-6|$

$= |-2| \cdot |x+3|$

$= 2|x+3|$

$< 2\delta = 2\left(\frac{\epsilon}{2}\right) = \epsilon$

4. $\lim_{x \to 1} 3x+1 = 2$

choose $\epsilon = 1$, assume $0 < |x-1| < \delta$

whenever $|3x+1-2| = |3x-1| < 1$

Then $-1 < 3x-1 < 1$

$0 < 3x < 2$

$0 < x < \frac{2}{3}$

This is not an interval surrounding $x = 1$ where x-values must be for $\lim_{x \to 1} 3x+1$

Therefore, this limit is incorrect.

5. $\lim_{x \to 4} x^2-4 = 12$

Proof: Given $\epsilon > 0$ choose $\delta = \min\{1, \frac{\epsilon}{9}\}$

Suppose $0 < |x-4| < \delta$

then $|x^2-4-12|$

$= |x^2-16|$

$= |x+4| \cdot |x-4|$

$< 9|x-4|$

$< 9\delta$

$\leq 9\left(\frac{\epsilon}{9}\right) = \epsilon$

also suppose $|x-4| < 1$

$-1 < x-4 < 1$

$7 < x+4 < 9$

$|x+4| < 9$

6. $\lim_{x \to 3} x^2 - 2x + 1 = 4$

Proof: Given $\epsilon > 0$ choose $\delta = \min\left\{1, \frac{\epsilon}{5}\right\}$

suppose $0 < |x-3| < \delta$

then $|x^2 - 2x + 1 - 4|$

$= |x^2 - 2x - 3|$

$= |x-3| \cdot |x+1|$

$< \delta \cdot |x+1|$

$< 5\delta$

$\leq 5\left(\frac{\epsilon}{5}\right) = \epsilon$

let $|x-3| < 1$

$-1 < x-3 < 1$

$-3 < x+1 < 5$

$|x+1| < 5$

7. $\lim_{x \to 1} x^2 + 1 = 2$

Proof: Given $\epsilon > 0$ choose $\delta = \min\left\{1, \frac{\epsilon}{3}\right\}$

suppose $0 < |x-1| < \delta$

Then $|x^2 + 1 - 2|$

$= |x^2 - 1|$

$= |x+1| \cdot |x-1|$

$< |x+1| \cdot \delta$

$< 3\delta$

$\leq 3\left(\frac{\epsilon}{3}\right) = \epsilon$

let $|x-1| < 1$

$-1 < x-1 < 1$

$1 < x+1 < 3$

so $|x+1| < 3$

8. $\lim_{x \to 3} x^3 - 10 = 17$

Proof: Given $\epsilon > 0$ choose $\delta = \min\left\{1, \frac{\epsilon}{37}\right\}$

suppose $0 < |x-3| < \delta$

then $|x^3 - 10 - 17|$

$= |x^3 - 27|$

$= |x-3| \cdot |x^2 + 3x + 9|$

$< \delta \cdot |x^2 + 3x + 9|$

by Triangle inequality $\leq \delta \cdot \{|x^2| + |3x| + |9|\}$

$< \delta(16 + 12 + 9) = 37\delta$

$\leq 37\left(\frac{\epsilon}{37}\right) = \epsilon$

let $|x-3| < 1$

$-1 < x - 3 < 1$

$2 < x < 4$

$4 < x^2 < 16$

$6 < 3x < 12$

9. $\lim_{x \to 2} 2x^3 + 1 = 17$

Proof: Given $\epsilon > 0$ choose $\delta = \min\left\{1, \frac{\epsilon}{38}\right\}$

suppose $0 < |x-2| < \delta$

then $|2x^3 + 1 - 17|$

$= |2x^3 - 16|$

$= 2|x^3 - 8|$

$= 2|x-2| \cdot |x^2 + 2x + 4|$

$< 2\delta \cdot |x^2 + 2x + 4|$

by Triangle Inequality $\leq 2\delta\{|x^2| + |2x| + |4|\}$

$< 2\delta(9 + 6 + 4) = 38\delta$

$\leq 38\left(\frac{\epsilon}{38}\right) = \epsilon$

let $|x-2| < 1$

$-1 < x - 2 < 1$

$1 < x < 3$

$1 < x^2 < 9$

$2 < 2x < 6$

10. $\lim\limits_{x \to 3} \frac{1}{x} = \frac{1}{3}$

Proof: Given $\epsilon > 0$ Choose $\delta = \min\{1, 6\epsilon\}$

suppose $0 < |x-3| < \delta$

then $\left|\frac{1}{x} - \frac{1}{3}\right|$

$$= \left|\frac{3-x}{3x}\right|$$

$$= \frac{1}{3}\left|\frac{3-x}{x}\right|$$

$$= \frac{1}{3}|-1| \cdot \frac{|x-3|}{|x|}$$

$$= \frac{1}{3|x|} \cdot |x-3|$$

$$< \frac{1}{3|x|} \cdot \delta$$

$$< \frac{1}{3}\left(\frac{1}{2}\right) \cdot \delta = \frac{1}{6}\delta$$

$$\leq \frac{1}{6}(6\epsilon) = \epsilon$$

if $|x-3| < 1$

$-1 < x-3 < 1$

$2 < x < 4$

$\frac{1}{2} > \frac{1}{x} > \frac{1}{4}$

$\frac{1}{|x|} < \frac{1}{2}$

11. $\lim_{x \to c} \frac{2}{x} = \frac{2}{c}$

Given $\epsilon > 0$ choose $\delta = \min\left\{ \frac{c}{2}, \frac{\epsilon c^2}{4} \right\}$

suppose $0 < |x-c| < \delta$

then $\left| \frac{2}{x} - \frac{2}{c} \right|$

$= 2\left| \frac{1}{x} - \frac{1}{c} \right|$

$= 2\left| \frac{c-x}{cx} \right|$

$= 2 \cdot |-1| \cdot \left| \frac{x-c}{cx} \right|$

$= 2|x-c| \cdot \frac{1}{c} \cdot \frac{1}{|x|}$

$< 2\delta \cdot \frac{1}{c} \cdot \frac{2}{c} = \frac{4\delta}{c^2}$

$\leq \frac{4}{c^2}\left(\frac{\epsilon c^2}{4} \right) = \epsilon$

if $|x-c| < \frac{c}{2}$

$-\frac{c}{2} < x-c < \frac{c}{2}$

$\frac{c}{2} < x < \frac{3c}{2}$

$\frac{2}{c} > \frac{1}{x} > \frac{2}{3c}$

$\frac{1}{|x|} < \frac{2}{c}$